

Chapter 1. Riddle of the Sphinx: The Mysterious Role of Mathematics in Physics.

In the ancient world there was a myth about a creature called the Sphinx. This creature was a strange hybrid between an angelic woman (sometimes even endowed with wings) and a powerful lion. If you want to feel for this creature, take a look at the traditional Tarot trump card for Strength. It shows a calm and beautiful lady taming a lion with her bare hands. You will also sometimes find the Sphinx depicted as a hybrid creature perched on top of the Wheel of Fortune card in many Tarot decks. Or you may go to Egypt and stand by the great pyramids. It is said that the Sphinx would pose riddles to people, and, if they could not answer the riddles, the Sphinx would then devour them.

Exercise: Do you remember the Sphinx's riddle that Oedipus solved? What might it have to do with Observer Physics?

It occurs to me that the Sphinx herself is an appropriate symbol for the mysterious riddle of how mathematics comes to be such a powerful tool in the study of physics. Mathematics is a beautiful angel of the Mind, and physics is a powerful beast that commands a World of dynamic motion and force. Beauty and the Beast. Beauty calmly and effortlessly tames the Beast.

How is it that the elegant beauty of a purely mental discipline can handle the wild physical world with such precision, confidence, and accuracy?

In short, why is mathematics so useful in the study of physics?

Mathematics is an abstract language for describing mental constructions. Why should it be useful in constructing models that describe the physical world and even predict its behavior? Is there a subtle link between the structure of the mind and the structure of the physical world?

Let us begin with an understanding of what mathematics is.

First of all mathematics is a language. What is a language? A language is a system for communicating ideas. In its bare bones a language consists of a vocabulary of basic elements, and a grammar or set of rules for organizing the vocabulary in different ways. The set of rules is really a collection of relations and operations. Relations are ways of linking vocabulary elements into expressions, and operations are ways of transforming one expression into another expression. So mathematics, at its basis, is not really about numbers. It is a communication system for describing various relationships and transformations of those relationships.

1. In a landmark article the linguist Charles Hockett ("The Origin of Speech", **Scientific American**, 203/3 (1960), 89-96) identified a number of design features that are involved in the construction of languages. Primitive communication systems (for example, various animal systems) have only a few design features. More

sophisticated systems add more design features. This is true for mathematics as well.

Let's outline the basic design features that are found in human language. I classify Hockett's design features into three categories of three features each. My definitions of the features follow Hockett closely, with perhaps some difference in how I view duality and arbitrariness.

Since human language requires a minimum of two interacting participants who agree on the meaning of a communication, it is ruled by what in mathematics is called a dyadic equivalence relation. Dyadic means a relationship between two participants. Equivalence means that the communication between the two parties is equal -- theoretically they both can agree and understand each other equally well, and share the same information. Dyadic relations such as equivalence have three properties: symmetry ($a = b$; $b = a$), reflexivity ($a = a$), and transitivity ($a = b$; $b = c$; $a = c$). In the following outline we show how the design features of any full-fledged human language, including any richly developed mathematical system, spring from these properties. This is by no means the end point of language development or of mathematical development, but it is a beginning that provides a rich enough framework for lots of exploring.

1. The Language Community

- a. **Interchangeability:** A symmetric language community of individuals exchanging information.
- b. **Total Feedback:** A reflexive system that allows a communicator to monitor his message, editing and refining it through techniques such as revision, iteration, and redundancy.
- c. **Traditional Transmission:** A transitive system for accumulating, preserving, and transmitting expressions generation to generation. Each individual is only born with language ability and must learn to use a specific grammar(s) and lexicon(s).

2. The Content (Semantics)

- a. **Displacement:** transitive displacements in various dimensions such as time, space, reasoning, extrapolation, and truth.
- b. **Semanticity:** a reflexive meaning inherent in communications. A creation means what it means.
- c. **Duality:** a lexicon that symmetrically maps signs to corresponding significants and vice versa by means of definitions. Definition makes an abstract element 'concrete.' Different semantic systems (interpretations) may result in different dictionaries. Definitions are fundamentally "circular".

3. The Message System

- a. **Arbitrariness:** a symmetric modality for shifting communication from medium to medium. (For expressing a message, one medium is as valid as any other, and the message may be translated back and forth between media. Hence the means of communication is arbitrary. This feature ranges from choosing among synonyms to choice of medium.)

- b. **Discreteness:** a vocabulary of primitive (often undefined), self-reflexive, and discrete elements for constructing expressions. The elements can be distinguished as separate from each other.
- c. **Productivity:** Syntax provides a productive set of rules for transitively generating a rich universe of expressions. These rules are operational procedures for establishing and transforming relations between discrete elements.

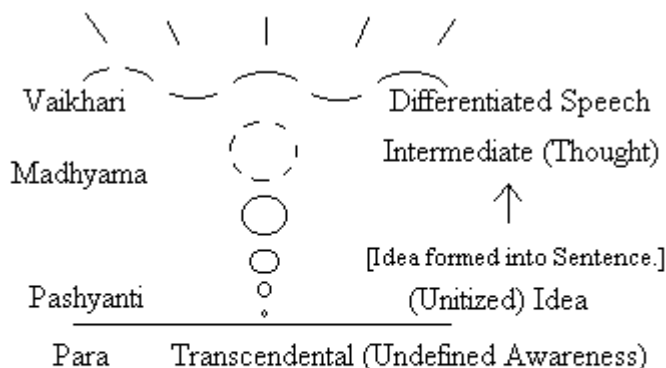
My use of the = sign to represent the dyadic relation is not exactly the same as a mathematical equivalence relationship. You can translate a sentence in language *A* into language *B*, and then from *B* into language *C*. The meaning will be the same as the sentence in language *A*. That is transitivity. So the features I call symmetrical (interchangeability, duality and arbitrariness) also seem transitive. But the key point with them is that you can go from *A* to *B* and from *B* back to *A*. The key point with Tradition, Displacement, and Productivity is that you can keep on going, transmitting from one to another. So when senders keep passing a message on, that is Tradition, not Interchangeability. When you keep translating to different languages, that becomes productivity, not arbitrariness. Any use of language tends to involve several or all of the features used all together at once, so the features overlap or bunch even though they are distinctly different features.

There is a tenth feature that seems a universal human feature. That is **Specialization**, the ability to perform other operations while carrying on communication. When a bee dances, his whole being is involved in communication. He can't do anything else. But we can hold a conversation while driving a car. This is a very useful talent.

Hockett mentions other features such as broadcasting, vocal-auditory channel, and rapid fading, but these features are characteristic only of special types of human communication and are not generally found all the time. They are not universal design features in human language, but are "localized". For example, speech fades rapidly, but not carvings on stone. Spamming involves broadcasting, but a private conversation (usually) does not. The vocal-auditory, broadcasting, and rapid fading features are actually sub-features of the choice of medium. Arbitrariness ranges from the choice of items to map in the lexicon to the medium that is chosen. Thus arbitrariness is closely related to duality, though still a separate feature.

It's fun and illuminating to examine various communication systems (including many animal systems) and identify what features they include. Mathematics, of course, qualifies as a full-fledged human language system. Numericity is not a separate feature in my opinion. It is a composite expression of several features.. For example, the generation of numbers in set theory is an exercise in productivity. The notion of set itself is an idea (semanticity). So is cardinality. Selection of symbols for numbers is duality. Numbers are written in an arbitrary medium and manner. The primitive elements of a postulational mathematical system exhibit discreteness even though they may contain notions of continuity.

From the above analysis, it is clear that the observer plays a fundamental role in the language of Mathematics. The observer not only creates all the components and expressions, he also determines all of the features he chooses to use and how he uses them. The design features may occur in clusters or all at once. According to ancient Indian linguistics the diverse features of language emerge from a unitized thought bubble called "sphota". Sphota emerges from "para", or undefined awareness.



Bhartrihari's Theory of the Formation
of Undefined Awareness into a Thought
Bubble (Sphota) that bursts into Speech.

A unique aspect of mathematics, similar to music, is that in its pure form it often has no specific interpretation or "practical world" semantic meaning. We say it is abstract. Pure mathematical objects are mental creations that are only weakly defined or wholly undefined in the physical world. This generalization power is deliberate and may be a combination of semanticity and displacement. Interpretations of abstract mathematical systems lead to models and applications, but such extensions are optional. The "meaning" of pure mathematics can be apprehended in an abstract, aesthetic way simply as patterns of relationships and their orderly transformations.

The "practical" usefulness of any mathematical system in physics comes from the creation of an interpretation that applies the mathematical system as a model for physical world phenomena. This is the process of mapping a message system to a physical situation. Physics further has a theoretical and applied aspect, each with its own mathematical regimen. Given these definitions, how can mathematics serve physics (and other sciences) so precisely and powerfully?

Mathematics is precise thinking. A mathematical system is a precisely defined and internally consistent set of abstract beliefs. Beliefs can be projected into experiences. The experiences are nothing other than strongly held beliefs. Therefore it is natural that we should be able to articulate in thought and language the beliefs that are expressed in the form of physical systems. The only discrepancy between a mathematical description and the corresponding physical system might arise due to transparent beliefs that have been overlooked or wrongly selected beliefs that are not consistent with the physical system.

A number, algebraic expression, or relationship can be very exact. Ordinary language is usually relatively imprecise thought. The previous sentence is an example, I suppose. So is that one. Let me show you how levels of CERTAINTY -- which correspond to levels of precision -- are reversed by the mirror (or perhaps lens) of observation that resides in the gap between observer and observed. The discrepancy with regard to certainty arises from an interaction between the mental principle of numericity and the physical quantum principle.

In the mental world of numbers, integers and rationals are discrete, carrying precisely certain values (infinitely precise?) and sequence. Non-algebraic irrational numbers, on the other hand are not precisely definable. We can not be certain what the n th decimal digit of such a number is. For example, we know that the n th decimal of 5 is 0, and the n th decimal of $1/3$ is 3. We can't do this for hardcore irrationals. They are fundamentally uncertain with regard to their decimal values.

Decimals can be viewed as infinite mental wave forms. Their component digits get "smaller" by orders of magnitude as they go off into the mental "distance". They can be written down symbolically in part. This part may be complete if the decimal "terminates" and then degenerates into an infinite string of 0's.

In the world of quantum physics, the basis for our world of objective experience, we find that the continuous wave function is precise and certain, but the discrete quantum particles that we observe are totally uncertain in the way that Heisenberg has described. This situation puts physics in a quandary when it uses mathematics to describe physical systems. We habitually use integers to count photons and electrons and such. But their locations and/or momenta (i.e. how we define them -- mass, velocity, position, etc.) are inherently fuzzy because of Planck's constant.

Wave functions tend to be continuous probability distributions spread out over time and space. Probability is often expressed by a value between 0 and 1. The wave function tells us the probability for a certain particle or event at any point in space or time. Most real functions are continuous. For example, here is a definition of a function from a standard calculus text (Edwards and Penney, 2nd ed., p. 9.)

* A real-valued function f defined on a set D of real numbers is a rule that assigns to each number x in D exactly one real number $f(x)$.

And here is how continuity is defined in the same text, p. 63.

* Suppose that the function f is defined in a neighborhood of a . We say that f is **continuous at a** provided that $\lim (x \rightarrow a) f(x)$ exists and, moreover, that the value of this limit is $f(a)$.

Functions may be discontinuous and not necessarily real-valued. Quantum wave functions are generally expressed in the complex domain. This is a set that contains the

reals as a subset. In any case any differentiable function must be continuous, although there are continuous functions (such as certain fractals) that are not differentiable.

Real-valued and complex valued functions are mostly comprised of irrational values, yet the evolution of the qwiff (quantum wave function) is fully determined and certain once the initial conditions are set. So we use discretely certain numbers to construct mental images of uncertain physical objects and fuzzily uncertain numbers to construct our mental images of physical systems that are certain. Most of us, including physicists, live our lives all twisted and misled by appearances.

Most people look at the world upside down through the funhouse mirror of the mind, where the simple counting numbers are supremely reliable. That is why people mistakenly put so much trust in physical objects. Integers are trustworthy beyond all time and space, and they also resemble abstract objects. So people usually "calculate" the objects in their world using trustworthy natural numbers, integers, and rationals.

We say, "I saw four chairs."

We do not say, "I saw 4.2968365128753274638923647830192003784.... chairs."

We count one apple, two apples, three apples. Tomorrow or next year the numbers 1, 2, and 3 will still be around, but those apples will be gone. People should really trust the qwiffs that they can't see, and not the apples. Of course, it's OK to talk about 2 apples if you are going to eat them today and then move on. So this rough mapping works well enough in a limited reference frame. But it breaks down as soon as you stretch the boundaries of the reference frame.

The qwiffs tell the evolution of the apple wave function over time. The particles that make the apples are only symptoms of the presence of qwiffs. When you count apples and oranges, you have an illusion of a reliable count because of the quantum statistics of so many atoms. But it is an illusion nonetheless. If you expand your frame of time and/or space that you observe them in, they get fuzzy real quick.

Look at an orange under an electron microscope. Leave it sitting in the sun for a few weeks. Where is it? The discoveries of quantum mechanics invite physicists to uninvert the mirror that they use to observe the world. Perhaps the qwiffs should be dealt with as rationals or algebraic numbers and the quantum particles counted as irrational "peanut" numbers.

I call irrationals "peanut" numbers because they are like the styrofoam peanuts used as packing fill. Mathematicians made them up to pack intervals until they became continuous. No mathematician can build continuity from the discrete components he starts with. Look at the definition of continuity again. The mathematician must use the concept of limits to convey the idea of continuity. This involves an infinite sequence. In the real world there are no infinite sequences.

Somewhere, at a superfine interval, the mathematician makes the quantum leap. He

gives up going to smaller and smaller neighborhoods of a , and jumps over to the limit. Here is the definition of limit given by Edwards and Penney, p. 41.

* We say that the number L is the **limit** of $F(x)$ as x approaches a provided that, given any number $e > 0$, there exists a number $d > 0$ such that $|F(x) - L| < e$ for all x such that $0 < |x - a| < d$.

So the concept of a limit is that you can sneak up on it by taking values of x that get as close as you like to a , but not equal to a . Then the corresponding value of $F(x)$ will get correspondingly close to L .

The tricky part is that the mathematicians like numbers to have no physical size in their geometry interpretation. They are infinitesimally small 'point-size' objects. The complete set of real numbers written as infinite decimals can be represented in a geometry interpretation by the points on a line segment between 0 and 1. (This segment can stand in for the entire Real Line. Real number sets have a holographic property that they reflect the whole in the part.)

The situation is complicated by the use of the word "neighborhood" in the above definition of continuity. I could not find a definition for "neighborhood" in Edwards and Penney, although they do speak of a "deleted neighborhood". So we define the foundation for calculus using a key term that lacks a definition. This is a general property of mathematical systems -- they are based on a small number of discrete primitive elements that are undefined. When you try to look squarely at them, they "fuzz out" completely on you. Furthermore, the notion of a deleted neighborhood (we don't allow to become exactly equal to a) is used to justify an expression that otherwise results in division by 0 at the limit when $x = a$, a situation that causes the function to explode into -- you guessed it -- INFINITE UNCERTAINTY.

* $F(x) = f(x) - f(a) / x - a$.

This is the basis for calculus -- infinite uncertainty. And yet it results in extremely precise and marvelous calculations. Very odd.

The invention of zero was a great advance in mathematics. It greatly expanded and facilitated the performance of arithmetical calculations. But there was a problem when you tried to divide by zero. To patch things up, division by zero was outlawed. What Newton did was show that this outlawing of division by zero was an overreaction. He broadened his viewpoint and generalized the operation of division by zero. In so doing he discovered that the problem of division by zero only really occurs in special cases. The triumph of this insight was his discovery of the calculus and a beautiful way of resolving the paradox of Zeno.

Draw a figure of any kind. Organize that figure by imposing an x-y coordinate Cartesian grid over it. We can now write an equation in terms of (x) and (y) to describe the figure. The operation of division can be represented as a ratio of the rise (Dy) to the

run (Dx) of any point in our figure in terms of values on our x-y grid. This ratio is called the "slope", since that is just what it looks like -- the slope of a terrain. It usually is a roughly triangular shape. Newton studied the behavior of these "triangularish" slopes and discovered that they more or less resemble right triangles. In general, the bigger the slice of the drawing we consider, the less they resemble the corresponding triangle. The smaller slices get closer and closer to the corresponding perfect triangle. The idea dawned that there could be a limit at which the two become essentially identical. That limit is reached when the run (Dx) is reduced to zero. Since the slope ratio is the rise over the run, this is like dividing by zero.

To his surprise Newton found out that reducing the "run" to zero did not cause the ratio to explode! It only "explodes" into an indeterminate value when you think of the denominator of the ratio in terms of a solitary cardinal number instead of an **interval** in an ordinal sequence. By just slightly shifting his viewpoint, he found that there is a whole class of possibilities that occurs in the limit of an infinitesimalized ratio, and only in one case does the ratio seem to explode, and that case makes sense too.

This was the value of visualizing a model. He could see that something was really there. If you inscribe a right triangle inside a curve and then squeeze its (Dx) interval closer and closer together along the curve, the diagonal hypotenuse that represents the slope of the triangle squeezes up closer and closer to the curve. When (Dx) reaches zero, the slope of the triangle becomes the tangent to the curve at the point where (Dx) converges. From this Newton discovered the principle of the derivative.

- * $y = x^3$; $dy / dx = 3 x^2 =$ a variable nonlinear slope.
- * $y = x^2$; $dy / dx = 2 x =$ a variable linear slope.
- * $y = x^1$; $dy / dx = 1 =$ a constant slope.
- * $y = x^0$; $dy / dx = 0 =$ a flat line with zero slope.
- * $y = n$; $dy / dx =$ undefined = infinite slope, no slope at all -- a vertical line.

These mathematical expressions all have graphical representations. Not only did nothing explode (except, understandably, in the last case), but Newton had found a way of determining an "instantaneous" or point value ratio in a varying situation. He also demonstrated the obvious fact that Achilles does beat the tortoise. The rest is history, and lots of textbooks full of formulas, as they say.

Experiment: Go outside of your house or apartment and stand at some distance from the front door. Leave the door open. (If the weather is inclement, do this indoors going from your living room to a bedroom.) Walk half way to the doorway. Stop. Now walk half of the remaining distance to the doorway. Stop. Now walk half the remaining distance to the doorway. Stop. Continue in this fashion until you find yourself standing on the threshold of the doorway right at the jamb. What prevents you from stepping inside?

This demonstrates the power of challenging a generally accepted assumption and moving in for a closer look.

What about the neighborhoods in which our function was defined? Hausdorff space is developed from two primitive **undefined** elements, *points* and subsets of these points called *neighborhoods*. In Hausdorff space the limit point is defined as follows:

* A point x of H will be called a limit point of a subset S of H provided every neighborhood of x contains at least one point of S distinct from x . (Eves and Newsom, **Introduction to the Foundations and Fundamental Concepts of Mathematics**, p. 260.)

From this definition of a limit one may derive the theorem that a neighborhood contains an infinite number of points. But when we look closer, we are left in a quandary, because the concept of *neighborhood* itself is undefined. This is a fundamental characteristic of **all** mathematical systems, however marvelous and useful they are. When you go down to the bottom of any mathematical system to see what it is made of, you find that it is built from at least **TWO undefined** elements. The house of cards is built on uncertainty. It arises from an undefined field. This is something worth exploring. And we will do that in some detail as we explore observer physics.

When we go down there to find the limit and we get into the "neighborhood", suddenly everything gets fuzzy. The fuzziness comes from the "Heisenbergish" uncertainty that is built into mathematics at the ground floor -- just like physics. In the system of real numbers this uncertainty shows up most clearly in the non-periodic irrationals.

The non-periodic irrationals make up the major "spatial portion" of the decimals between 0 and 1 on the Real Line Segment. Because points are size-less, the irrationals must therefore HAVE FINITE "GAP" SIZE. Otherwise you will never get anywhere in filling the space between two 'points' no matter how many infinite buckets of points you throw in. But that gap size is indeterminate. There is no standard for it. It is what in Hausdorff space is called a "neighborhood". Lines are made of points and certain subsets of points called neighborhoods. The points are the rationals and the neighborhoods are the irrationals.

The observer defines the SIZE of a neighborhood.

Exercise: Here is a simple exercise to help you understand this principle.

- (1) Mark two dots an arbitrary distance apart on a blank sheet of paper.
- (2) Add a dot anywhere between the two starter dots.
- (3) Add more dots anywhere between the three dots.
- (4) Continue this process until you begin to see a line manifest between the two starting points.
- (5) All the dots represent point-value integers and/or rationals and/or algebraic numbers.
- (6) The spaces in between represent neighborhoods.

Theoretically those two original points determine a line, but the line is as yet unmanifest. Your mind fills in the line. The dots become a line only when you **believe** they become

a line, and not until then. This is **observer physics**. The observer controls the whole process and determines how and when the "quantum leap" is taken. That leap is a leap of faith. The dots become a line only **because** you believe they form a line, and only **when** you believe they form a line. With practice you can shift your viewpoint and see the line once again as a set of dots.

So the process of the "quantum leap" is **reversible** and thus **symmetrical**. In fact we have just done the reverse of the usual quantum leap in physics when a continuous wave jumps to form a dot-like particle!!

We assume that the dots can be infinitesimally small. But this is not true in the 'real' world. Even thoughts occupy mental space and require energy. We are stuck with the situation that at some subjective point our mind fills in the line from the stand point of perception, like a TV image getting smoothed out. Instead of dots we see a line. Do the steps outlined above and notice when the dots seem to connect in your mind to give the impression of a line. This is the game of "Connect the Dots." Can you "disconnect" the dots?

If the dots are finite, then we don't need "neighborhoods" to fill the gaps. We just fill the space with dots. If the dots are size-less, then we need "neighborhoods" to fill the gaps. But as long as we can distinguish one dot from another, there must be a gap between the dots.

This is the principle of the **GAP**.

In his course called **The Science of Creative Intelligence**, Maharishi emphasizes the notion of a gap that sits between any two creations. Palmer also discusses this principle in his **Avatar Materials**. The notion of a gap is essential to the notion of **continuity**. Dots and gaps are complementary. The gap has indeterminate size. It is undefined. The "ungaps" -- or dots -- on the other hand, are well defined, but have no size. Or you can switch them around. Generally we stick in an arbitrary number of gap numbers to fill in the gaps.

Cantor liked to think of the set of irrationals (gaps) as hugely infinite, "uncountable", and infinitely more numerous than the infinity of rationals (dots). We can just as easily envision a gap between each dot. However many dots we have, finite or infinite, determines the number of gaps. The value of a dot "labels" the adjacent neighborhood gap peanut number, giving it a precise value. We can label a dot as (Na), and its partner gap as (Nb). For every (Na) there is a corresponding (Nb). Thus we find exactly as many gaps as dots, minus one, the terminal point, which has no partner.

You decide how many dots and gaps there are. Minimum: 2 dots and one gap. Maximum: "infinity" dots and infinity minus one gaps. There's always one more dot than gap for a line. You need a terminal at each end of a line segment. See how simple it is?

However, one thing is certain. Gap numbers are fundamentally different from dot numbers, just like quiffs and particles are different animals. Yet they are inseparable like the two sides of a coin. They have a dependent relationship. Gap numbers are "connect-the-dot" numbers. They can't exist without dots. They are dot-dependent. They are countable because each gap is labeled by its contiguous dot. If the dots are finite, the gaps are finite. If the dots are infinitely many, the gaps are infinitely many. The gaps have no fixed size. The observer determines their sizes. This is his metric.

Cantor formulated a clever proof that the irrationals are uncountable. We'll look at it in more detail later. In framing this proof Cantor turned up a paradoxical problem in math akin to Goedel's incompleteness theorem, because the purpose of numbers is to count. Cantor's proof by contradiction begins by supposing a theoretical list of all the irrational decimals. A list is by nature countable. Then Cantor "diagonalized" his list, changing digits in such a way as to generate a new number in the set but not on the list. Thus he showed the "uncountability" of the set of real numbers.

It seems very strange to say that irrationals are a kind of number you can't count with. How do you know which is bigger and which smaller if you can't write them in full? You can't count them ordinally as a set or even write a single one -- and yet you need them to do math! This smacks of contradiction. This situation is like proving that yes is no or that $1 = 0$ in a binary number system.

The interpretation of the irrationals that I propose -- gap numbers or "neighborhoods" -- posits them as REAL real numbers. They map one-to-one (minus one if you include both terminals) to the rationals, and are countable, and behave like normal numbers. But, like imaginary numbers, they are in a different mental dimension of reality from the rationals. Yet the two different mental dimensions are nicely connected, complementary, in a one-to-one relationship, and you can jump from one dimension to the other with no problem. Continuity becomes a non-issue. The role of the Observer becomes a major issue. The Observer defines the resolution of the system, the size of the neighborhoods, and thereby the threshold of continuity, the stage at which the collection of points quantum leaps to become a continuous line -- or, vice versa, the continuous line quantum leaps into a collection of dots.

Redo the "Connect the Dots" exercise and pay close attention to the moment when you suddenly see the set of dots as a line. Does the threshold change when you do the exercise more than once?

The old children's game of "connecting the dots" is a fundamental game of quantum physics. The dots are particles, and the finished drawing is the quantum wave form. Once we understand this game, we will understand the transduction of energy, (how the forces fit together), and the vital role of the Observer in the whole scheme.

The role of the observer is clearly one of decision and interpretation. These are judgmental processes that introduce bias into a system.

Any element of a system may have several, or even unlimited, different interpretations. We may interpret an element as a graphic element, a symbolic semantic element, a numeric element, a relationship link, a transformational operator, and so on. How an element is used is an arbitrary choice of the scientist who is organizing and applying the system. Some believe that the numerical interpretation of elements -- counting ability -- may even qualify as a design feature of human language. I suspect that it is merely another name for the feature of productivity.

Question: If you examine the data in a computer, you find it consists of nothing but arrays of numbers in the form of 0' s and 1' s. Yet some of these 0' s and 1' s represent numbers, some represent words and messages, some represent graphics, and some represent program instructions. How does the computer know which is which? Who decides that?

Numbers as numbers can be interpreted in various ways. For example, they may be cardinal or ordinal. Cardinality refers to a certain quality of a set, and ordinality refers to the relation of one set to another. Even more primitive than number may be the notion of a set, a collection of objects. Some mathematicians derive number theory from set theory.

What is a number? We might say that a cardinal number is a label placed on a set. A set is a countable collection of similar objects. The number label indicates uniquely how many members there are in the set, rather than any other properties assigned to the members. Thus cardinality is the answer to the question, "how many?" Ordinality is a linking of numbered sets by a relation such as "is less than" (<) or "is greater than" (>). Cardinality is an expression of the design feature of discreteness, and ordinality is an expression of the design feature of productivity. Notice how we defined number in terms of "countability". How do we define countability? It is the ability to place labels on sets to represent the number of members in each set. We are circular. Circularity of definitions implies the principle that at its basis every language (including mathematics) begins with undefined elements.

My **American Heritage Dictionary** defines "count" as "to name or list (the units of a group or collection) one by one in order to determine a total." A "label" is "an item that serves to identify....a distinctive name...." "Number" is defined as "one of a series of symbols of unique meaning in a fixed order that can be derived by counting." "Symbol" is defined as "one that represents something else . . ." "Represent" is defined as "To stand for; symbolize."

At some point we just have to **believe** that we know what we are doing when we are "counting" with "numbers".

Graphics connect with sensory images by an imagined resemblance. Linkage appears as juxtaposition in time or space or other dimensions. Operators suggest processes of physical transformation and change. Symbols, by conventional agreement, represent personal choices and interpretations. Numbers express an experience of sets of objects

that are discrete, yet share common properties within the set. The number label represents the notion of how many members are in the set.

Most natural languages have words for objects that are countable and for objects that are not considered countable. For example, 'table' is a countable noun, and 'milk' is a non-countable noun. We can have five tables, but not five milks. We can have five 'cartons' of milk, though, because the carton is a container for the weakly bounded object, 'milk.' We can have five cases of cartons of milk. So cardinality of sets has something to do with the boundary that defines an object and gives it discreteness. Usually, but not necessarily, this is in time and/or space.

Mathematicians have studied the types of numbers that can exist under the ordinary arithmetic operations. The most primitive numbers are called the natural numbers, N . They are what we call the whole numbers. They are the set of numbers related by the "size" dyadic relations, $(a < b)$ and $(b > a)$ and $(a = b)$. Then come the integers, Z . They are a set related by the binary operators $(a + b)$ and $(a - b)$ as well as the size relations. Then come the rational numbers, Q . They further include the binary operators $(a \times b)$ and (a / b) . Then come the irrational numbers, R , and the imaginary numbers, I . They still further include the operators $[(a)^n]$ and $[(a)^{(1/n)}]$. For example, $2^{(1/2)}$ is irrational, and $-1^{(1/2)}$ is imaginary. With these sets we have sufficient operators to do all of ordinary mathematics. We also have numbers that correspond to every relation and operation defined in the system.

Mathematicians along the way developed a shorthand way of representing numbers: the decimal system. The invention of the decimal format provided a very convenient way of writing numbers. The problem was that many rational and irrational numbers actually produced infinitely long decimals. The rational numbers at least produced periodic decimals, so you could write out the period, and then just add a symbol indicating that the period repeated itself indefinitely. The irrational numbers were nowhere nearly as cooperative. They formed non-periodic decimals. So there was no way to indicate their exact value in decimal format. What was worse, mathematicians showed that, in addition to the algebraic non-periodic decimals, there existed infinitely many non-algebraic non-periodic decimals representing other irrational numbers.

Because you could not write out their exact value even in principle, you could not even count them. They were uncountable. This created an awkward situation. Numbers originally were defined and intuitively understood as representing the countable nature of objects. Sets were supposed to be countable. But here were numbers that no one could count in any precise order no matter how hard they tried! They were uncountable in principle.

Were these "peanut" numbers really numbers? Well, as we have seen, there is a whole class of physical objects defined in natural human language that are inherently uncountable. These include such common items as water, milk, butter, mud, cement, and so on. Maybe there is a connection between uncountable objects and uncountable numbers.

Mathematicians went further and discovered that the algebraic description of the world of numbers corresponded exactly to a description of the world of shapes using geometry. The only difference was one of interpretation and the medium of expression. Thus was born analytic geometry and the correspondence of numbers with points on lines and in space. This invention of Descartes' was a major step forward in the development of modern science.

But the decimal system, while compact and efficient, has yet another awkward feature beyond the introduction of nonperiodic decimals. Any infinitely repeating decimal that turns into an infinite string filled with the highest primitive digit in a particular base (i.e. 9 in base 10, and 1 in base 2) is equivalent to another decimal with an infinite string of 0' s. That results in two numbers in the notation system mapping to one single value. The real numbers contain infinitely many such duplicate numbers. This is not a good situation when each unique number is supposed to have a unique value. To fix this situation, the mathematicians made a special ad hoc rule that disallows infinite strings of 9' s in base ten and disallows infinite strings of 1' s in base two. When they found that the relationship of any number divided by 0 explodes, they simply made another ad hoc rule disallowing 0 in the denominator of a rational number. These ad hoc rules seem to handle the problems with the number system, but they end up marring the elegant symmetry that we intuitively seek for mathematical notation. Zero and infinity turn out to be especially tricky components of a mathematical system. Sure enough, they turn into challenges for the physicist as well.

In any case, every algebraic expression could be translated into a corresponding expression as a graphic image of geometry. The theory of functions was born. The role of the uncountable irrational numbers in the geometry interpretation was that they provided continuity between the discrete natural numbers, integers, and rational numbers. It was a very challenging proposition to prove the continuum hypothesis: that the decimals form a continuum just like a line appears to form a continuum. Mathematicians such as Dedekind felt they had achieved this, although Goedel subsequently showed that the "proof" of continuity of the real number system is inherently a postulate independent of the rest of the postulates of the real number set (rather like the parallel postulate in geometry). In other words, you can take it or leave it, just like your choice of geometry can be Euclidean, or Riemannian, or Lobachevskian, and so on.

Goedel also showed that the real number postulate system is incomplete, and it is not possible to prove the real number system to be consistent by methods belonging to the system itself. Notwithstanding these somewhat disillusioning discoveries, the real number set still provides a nice analogy to the points on a line. It also fits pretty well with the general intuitive notion of continuity that we seem to observe in space-filling objects that lack self-defined boundaries. The uncountable irrational real decimals play the role in mathematics of the space, air, water, cement, and mud of our "real" world. They can be said to exist as objects, but they are "uncountable". You can't say for sure their exact value. Uncountables may also serve for abstract notions such as

"consciousness," "sleep," "sorrow," "happiness," "intelligence," and so on. If you can give it a label, but can not say exactly where it is or how much it is without adding other definitions (such as a bottle), then it is uncountable. It is a gap number. Just like the above notions from our "real world", it fills in the gaps between discrete objects.

So we seem to have a handy mapping between mathematics and our world. The world of numbers and geometry can paint a pretty accurate, sometimes amazingly accurate, picture of our world. We explain this as a reflection of the principle that the physical world is an expressed form of a set of mental beliefs. But then there is the issue of predictability and certainty. These are key design features of science. The discovery of fundamental uncertainty in physics by Heisenberg precipitated a major crisis, a blow to the belief that it was possible to describe all the parameters of a system with unlimited precision. Heisenberg showed that the parameters were arranged in a conjugate fashion that disallowed precision for all of them. You have to make a choice of what you want to look at closely.

In the world of mathematics, numbers are the objects. The natural numbers generate a set of precisely defined discrete objects. The ordinal value of each natural number is entirely predictable. From any given natural number, we can always generate the next number in the sequence by using the productive rules for number generation. But maybe this is just our choice. Maybe we have a transparent belief that natural numbers, or any other numbers, can be precise. Maybe you choose dot numbers to be precise and gap numbers to be fuzzy or the other way around, but not both ways at once. Maybe there is yet another way of looking at it.

Numbers can be mapped to physical world phenomena so that natural numbers and algebraic expressions describe actual objects and processes. Unfortunately as we shift into the quantum view of things, we discover that even apparently very discrete objects such as protons and electrons lack absolute precision and take on quantum uncertainty.

For example, suppose we direct a stream of electrons with consistent velocity through a tiny aperture to collide with a screen that registers each collision. Intuitively we expect the electrons all to strike the target at the same spot. In fact we find that they are distributed randomly over the screen in a wavelike pattern called an Airy diagram.

On the other hand, we can describe mathematically the continuous time evolution of a quantum event by means of a wave function. The function that describes the Airy diagram has the liquid wavy quality of vibrating air or water and forms a resonant pattern that fills space and time. This property of continuity resembles the real number set. But the certainty aspect of the wave function resembles more the natural numbers. At any arbitrary moment or position we know the precise value of the function and the shape of the wave. But we are totally unable to predict where any particular electron will strike the screen, just like we are totally unable to predict what the n th digit of a nonperiodic decimal will be. We do know that the electron will tend to fall at the more probable areas of the wave function's pattern on the screen.

So there is a lens-like operation that goes on between the mental world of mathematics and the physical world of phenomena that reverses the appearance of certainty. We suspect that this is caused by a property of the physical nervous system. What we call the "nervous system" is the medium connecting the mind and senses to the physical world. We may have uncovered a previously transparent feature of human belief systems. The two sides of the lens match in terms of 'shape' but are reversed in terms of the assignment of certainty. This is exactly analogous to the way a visual image gets reversed spatially by passing through a lens or by reflection in a mirror. It is also analogous to the way a light field gets reversed in time when reflected in a conjugate mirror.

So our first major new discovery of Observer Physics is this:

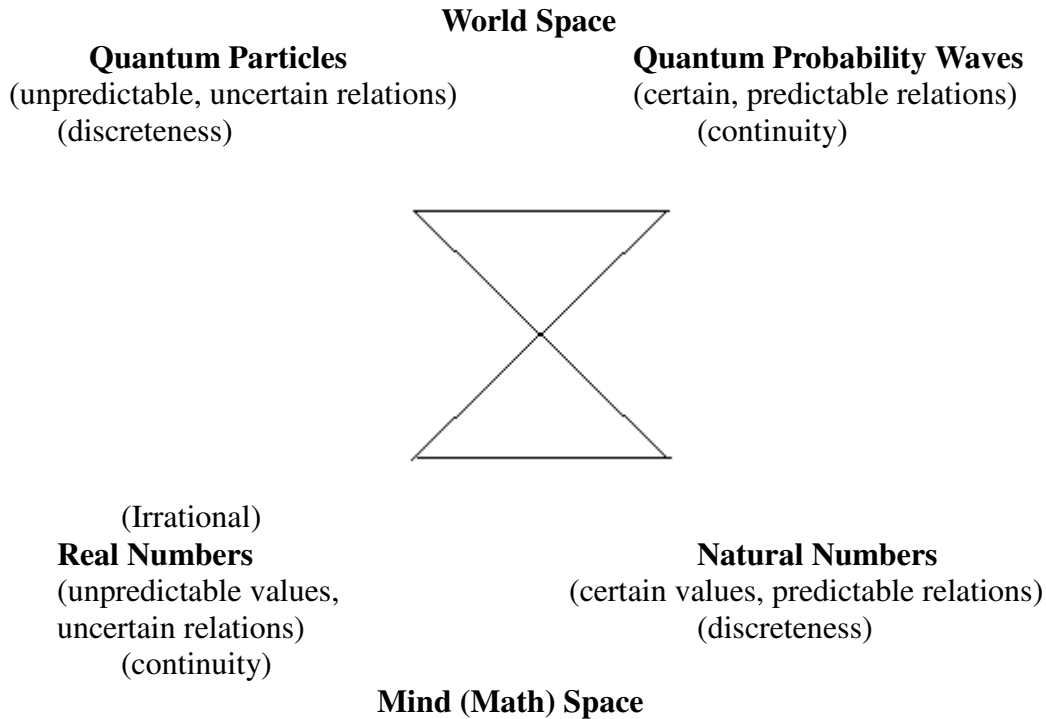
The observer's Mental Space has a relation to his physical World Space characterized by a reversal of certainty.

It is not that the observer turns things spatially or temporally upside down in his mind from the way they "really" are. That may happen also, but we can not prove it, because we only see what we see. Influenced by the transparent effect of the reversal of certainty, people look in the "wrong" place in the physical world for the stability and certainty they know and expect in the mental world. They try to build a stable, logical world out of discrete physical objects based on the analogy of their mental world's discrete and logical natural numbers. These natural numbers seem to fit so nicely to the objects they observe. Unfortunately, the physical objects are inherently non-local energy fields that precipitate into random and unpredictable particle events when observed, so this house of cards is built on sand and subject to the breeze and the tide. Alas, we seem to live in a physical world of entropy where randomness rules the roost.

Paradoxically, if someone wants real stability, he must turn to the uncountable aspect of his world: the unbounded non-local flows of air, water, space, consciousness, and so on. These are described by the wave functions and exist physically only as patterns of huge statistical probability ensembles that shift about according to the time evolution of the wave function. Non-local flows exist forever as waveforms pulsing throughout the universe.

A single molecule of water does not make water. Water is the manifestation of the statistical outcome of billions of randomly distributed molecules, but it obeys the laws of fluids. We will explore this "reality" more deeply in our subsequent discussions.

On the next page is a sketch of the relationship between the World Space of Physics and Mind Space of Ideas.



This crisscross structure has a fractal nature. As Mandelbrot, Wolfram, and others have pointed out, in the Mind Space it is possible to generate highly complex and even totally random results from very simple algorithms. It is also possible for a randomly initiated algorithm to collapse into a discrete, ordered system by means of an inherent attractor feature in its design. Likewise, in the World Space, objects that appear to be discrete particles tend to expand and dissolve their boundaries under the influence of entropy or wave packet destructive interference until they become continuous wave functions with no discernible particle nature. On the other hand, any World Space wave phenomenon tends to have an attractor such as gravity or an observer's measurement that can collapse it into one or more localized phenomena.

Therefore the use of mathematics in the study of physics may lead a student to realize how he has been confused by the deceptive reversals of predictability and certainty between Mind and World. A natural number exists forever in Mind Space. A natural object is here today and gone tomorrow in World Space. The flow of uncountable objects such as water, air, space, and consciousness reliably fills in gaps to give the World Space continuity and certainty. The "gap" numbers that fill in the Mental Space have no precise values. They are optional, randomly structured, and the observer must arbitrarily define their values. Yet there are crossover methods in each space and between each space. Attention to how these methods operate eliminates confusion.

There is an observer viewpoint in which a perfect match occurs between the Mind Space and the World Space. The World Space IS the Mind Space, and the Mind Space IS the World Space. This is sometimes called Unity Consciousness. We will consider such a possibility more formally in Chapter 6.

Summary:

- * Mathematics is an excellent tool for modeling physical phenomena.
- * However, the physicist must be aware of what a predominant role the observer (physicist as mathematician) plays in the conception and operation of mathematics. This is clear from a careful contemplation of the design features of mathematics.
- * He must also be aware of the ' weak points' of mathematics, especially with regard to 0 and infinity. These values are also tricky to handle in the physical world. The basic elements of any mathematical system are **always** undefined. Thus any interpretation of the system is an arbitrary act by the observer, just as the very definition and construction of any mathematical system is an arbitrary act by the mathematician.
- * Another special aspect of the relation between mathematics and physics is the ' reversal' of predictability/certainty between the mind and the world. The predictable mental elements (natural numbers) map to the unpredictable physical objects (the quantum particles); and the unpredictable mental elements (real numbers) map to the predictable (continuous statistical wave functions) aspect of physical phenomena. Care must be taken because the properties of certainty with respect to similar appearing objects tend to be reversed when we map from mathematical objects to physical objects.
- * This crossover effect, or mirror-like inversion, is echoed throughout both mathematics and physics and forms a main theme of this book.

Sample Dialogue Using Palmer's "Transparent Belief Exercise".

See Exercise #23, **ReSurfacing** (Altamonte Springs, FL, Star' s Edge, 1997.)

Mathematics appears to provide a precise mapping of the physical world into the mental world. Why do our mathematical models break down at some point whenever we try to apply them to the world?

Qa. What might someone believe to experience a breakdown of mathematical models when applied to physical systems?

A. We are not working hard enough to find the right model.

Qb. What evidence supports this belief?

A. People who work hard get more accomplished. We have made much progress.

Qc. What other belief might someone have to experience a breakdown of mathematical models when applied to physical systems?

A. We must be more precise in our calculations and measurements.

Qb. What evidence supports this belief?

A. Precision allows us to refine our models. Successive paradigms have used better mathematical models and achieved more precise verifications.

Qc. What other belief might someone have to experience a breakdown of mathematical models when applied to physical systems?

A. Aha! Our wires have been crossed. At a fundamental level we are trying to force physical objects to behave like mental objects when they live in a different realm by different rules.

Qb. What evidence supports this belief?

A. In number theory natural numbers have certain values and irrationals have uncertain values, but in physics quantum objects have uncertain parameters and only the quantum wave functions are certain. The sharp definition and certainty of natural numbers and the poor definition and certainty in the value of non-algebraic reals are all well attested in number theory. All experiments to date support Heisenberg' s uncertainty principle regarding the fuzziness of particles and clarity of wave functions. This explains the paradoxical feeling we have that mind and body are somehow separate or different even though they are tightly bonded together.