Comments on Wolfram's
A New Kind of Science

Wolfram, Stephen. **A New Kind of Science**. Champaign, IL: Wolfram Media, Inc., 2002. This is another blockbuster book by Wolfram, the creator of **Mathematica**. The title is ambitious to say the least. The systematic study of cellular automata -- the main research tool used by Wolfram for his book -- may well become a new scientific discipline. But Wolfram's vision is much broader. He feels he has identified some global principles, and he is deeply exploring the boundaries and interactions between the mental (mathematical) and physical worlds. This makes him a major pioneer in observer physics.

A key theme of Wolfram's work is what he calls the principle of "computational equivalence" -- the ability of mathematics to mimic the physical world. If I understand him correctly, he means by this idea that as a system grows in complexity, there is a threshold beyond which all systems behave the same. (Does this mean that God -- and the aliens -- are no smarter than we are? Is this anthropomorphic jingoism? Or does it just mean that everyone is equally good at making a mess?) Perhaps Wolfram is talking here about the limit where complexity becomes pure randomness. Wolfram's idea of "Computational Irreducibility" refers, it seems, to the notion that at some point any model of a system essentially reproduces the system in another medium with the same complexity. Related to this is the mathematical notion of "universality" -- that a certain program can be set up to emulate a whole class of programs. A similar idea is the notion that you can emulate algebra with geometry and vice versa.

Wolfram's starting point in the creation of his "New Kind of Science" was his discovery that very simple programs can produce great complexity and even randomness. This is not really a new discovery. Mathematicians have known this since the discovery that a simple ratio such as pi is an irrational quantity that generates a decimal with an infinite string of random digits. Linguists are familiar with ways to generate randomness (or at least great complexity) from simple grammars, and more recently the fractal and chaos people have made a great deal of progress generating infinite complexity with simple mathematical structures such as the Mandelbrot set. (In Observer Physics we discuss the example of the growth equation with the Verhulst factor included so as to make the system nonlinear.) What may be special about Wolfram's approach is the importance and generality he attaches to this principle. Its flowering as a principle definitely is a product of the computer age. But, although Wolfram may not be the first to notice that a new kind of science is emerging, he is definitely one of the significant pioneers in this new science.

Wolfram has developed some wonderful ways of systematically exploring the behavior of computer programs. This is an extension of what people were already doing with computers to model behaviors of systems. But Wolfram has carried it to a more abstract and general level as a discipline in its own right. Like Mandelbrot in his breakthrough fractal research Wolfram uses the computer's powerful graphics capability to explore simple program structures, exploiting the human capability to analyze data in parallel through visual scanning of patterns. Combining this with the computer' s high-speed iterative power gives awesome results. He creatively uses the identity of geometry and algebra the same way that Mandelbrot and others have to speed his research and make it more accessible. Pictures are worth a thousand words.

Wolfram has come up with a general typology of system behavior with regard to its most general level of organization. He identifies four classes. There are really five classes of system "equilibrium" that show increasing levels of complexity, but Wolfram combines the first two. He really should keep them separate, because perfect homogeneity and total random mixing are the opposite limits of a spectrum. Also, for a nice symmetry in his taxonomy, he should divide Class 4 into periodic islands and nested islands.

Class 0:	Homogeneity without change (total simplicity)
Class 1:	Repetition (periodicity, oscillation); more generally -- "linking"
Class 2:	Nesting (periodicity with embedded structures); fractals
Class 3:	Randomness (chaos)
Class 4a:	Local Structures (chaos with local islands of repetitive order)
Class 4b:	Local Structures (chaos with islands of nested structures)
Class 4c?	Local Structures (chaos with local islands that migrate)

Classes 0, 1, and 2 exist in blank (homogeneous) environments. Classes 3, 4a, and 4b exist in chaotic (random) environments. Of course we can also add mixing of repetition and nesting at both levels (homogeneous and chaotic). There may also be a further distinction of inanimate and animate creatures at the outer regions of both levels unless such phenomena only occur in Class 4. We also do not know if animate forms are endemic and/or exclusive to class 4 or not. But obviously they do not appear in class 0, and, depending on point of view, may not appear in classes 1 and 2. For example, vertical stripes can be seen as periodic and nonmoving, whereas the same stripes tilted diagonally can be seen as migrating "non-locally" in the same way as the migrating islands of class 4. Plaid squares tilted form crisscross hatches like Rule 110. These can be regular or irregular. If we have a random field of black and white squares, presumably we could find a viewpoint that would neatly separate the two into two nice classes. For example, if you mix salt and pepper in a jar, you can add water, dissolve the salt, strain out the pepper from the salt water, and then allow the water to evaporate, leaving the salt and pepper nicely re-separated. The water acts as an attractor that shifts a class 3 system into a class 1 system.

Wolfram notes, as the chaos people do, that randomness can simplify by means of "attractors". Thus reversibility is possible for all systems, but exact path reversibility only occurs in periodic, nested, or pseudo-random (not pseudo-periodic or chaotic) systems. Homogeneity and randomness both wipe out path information, but you still can go back and forth from homogeneity to randomness. You just have multiple possible pathways -- a fairly obvious condition.

Wolfram has noticed that Class 4 systems have the interesting property of non-local communication via animated local orderly structures that can move about and interact with other parts of the system. If Wolfram's conjecture that all class 4 systems have animation is true, that is a major contribution. Unfortunately he does not prove it. He just gives some good examples and makes a conjecture. Rule 110 sometimes produces graphics that look like rigid scattering diagrams. Maybe this is the primordial source of the archetype for the "animal". Some automata just sit still like rocks. Some grow like plants. And some range about like beasts, or maybe just wandering asteroids.

Wolfram's classes are not new. Perhaps his assertion that they are universal and general is new. For example, in grammar the blank page represents homogeneity -- we call it "writer's block". Conjunction is repetition of a grammatical structure, usually with linkage by "and" or "but", or perhaps just with commas. Sentences are periodic grammatical structures. Essays are fractally organized into a theme with chapters, paragraphs, sentences, phrases, words, and letters. So language is highly nested with embedded structures. In literary circles randomness is usually referred to as "creative writing". You never know what the writer is coming up with next. This brings up another dimension to randomness. Just typing random letters or words is NOT creative writing. There's a difference. Creativity comes from unexpected, unpredictable shifts to different levels of awareness or points of view. You can have particles scattered randomly all over a screen, or you can choose a topic and view it from many different random viewpoints. In other words, the screen is the theme and the particles are the various viewpoints. Local structures come up in language as the use of refrains, asides, digressions, and parenthetical expressions, and themes within a complex framework. In my discussion of decimals I give examples of these various classes in number theory. Wolfram might take a look at Hockett's famous article in **Scientific American** (203/3, 1960): "The Origin of Speech." Particularly relevant are the features of productivity and arbitrariness. You can find a summary of this classic article on the Internet. (Also see **OP**, chapter 1).

In Chapter 11 of **OP** I discuss invariance, starting with the notion of the Hamiltonian and the conservation laws, symmetries, and invariances of physics. Embedded in that discussion is a long section about simple mechanical models such as cams and ratchets that describes a whole range of techniques of using symmetry breaking and phase locking to form localized equilibrium states embedded in larger systems. The range goes from complete homogeneity (obviously self-reversible), to periodicity, to nested structures, to thermodynamically randomized systems, to locally phase locked systems. I also give models of quantum techniques for shifting from one style to another or moving from one localized island to another, including also techniques for preventing island shifting -- non-local or nested phase locking.

Perhaps one of Wolfram's key contributions to observer physics is his discovery that with computer experiments you can deliberately **objectify** a precise mental-mathematical expression as a computer program and then systematically **observe** the behavior of the mental construction as a physical phenomenon via the computer output (p. 109.) He thereby extends his defined ideas step by step into physical forms. This is important, because, as debuggers well know, some programs behave in unpredictable ways that one would never guess without exploring the outputs of the programs at some length.

However, just as Einstein's principle of equivalence needs to be "flipped" to "conjugate equivalence", so also the principle of computational equivalence needs to be "flipped" by the mirror lens of certainty/uncertainty. (See Chapter 1 and Chapter 6 of **Observer Physics**.) The outputs of Wolfram's cellular automata programs may seem like mathematical structures, but computer printouts really follow the rules of quantum waveforms. As Wolfram extends the Mental Space into the World Space, the question then arises -- just where is the crossover point between mind and matter? Wolfram's techniques are reminiscent of Doug Henning's sliding knots -- he is sliding the Mind-World crossover further in the World direction. A program such as TM allows one to slide it in the other direction, and Avatar allows you to slide crossovers in any direction you like.

There is a clear threshold at the transition from homogeneity to incipient complexity (i.e., simple systems that modulate). Then there is a continuum of increasing complexity that is only defined by the observer. Then there is a "quantum leap" from complexity to "chaos". Randomness is the limit of infinite complexity -- no rules at all, though you can embed rules and hierarchies of rules within that background of no rules.

Wolfram writes in great detail about randomness and complexity, but only on p. 552 does he get around to probing for a good definition of randomness. After pecking at it lamely for a few pages, he ends up with the notion that when something is random, no "simple program" can "detect any regularities" (p. 556). **The irony of this definition arises from his claim that simple programs can generate randomness and complexity.** He asserts, but does not prove, that no simple program can detect any regularities in the object of examination. Also he is not giving us a "randomness" test. He's talking about a "regularity" test. Where is his "randomness" test? If I'm right about randomness being a limit at infinity, then there is no randomness test. Randomness is non-local and infinite. It is the field of undefined awareness, the field of all possibilities. You experience that when you transcend in meditation. Maybe transcending is the only test of randomness (see below).

Wolfram notes the apparent similarity of complexity to randomness, and after whacking at that notion for a while, he arrives (p. 559) at the idea that something "seems" complex if we can't extract a "simple description" of it. **This again is ironic in the light of his demonstration of how very simple algorithms can generate extremely complex behavior.** Isn't his thesis telling us that, for example, **Rule 30 is a brief, but complete and precise, description of a certain style of complex, or perhaps even totally random, behavior**? Why doesn't he come out and state clearly that randomness is the limit to complexity? If this is so, then the outputs of any two programs that generate total randomness are essentially identical. If the threshold for the onset of true randomness is at infinity, there may be no finite test for it. On the other hand, the onset of complexity (from simplicity) is finite, but subjective (**observer-defined**.) Palmer (**ReSurfacing**, p. 5) suggests that **something seems complex because it doesn't fit in with what you already believe.** He means by this that complexity is observer-defined.

A math professor finds calculus simple, but a grade school student finds it complex. Eventually, after a bit of indoctrination, the student may also find calculus simple. Or he may put some attention on it and figure it out, and then find it simple.

Wolfram asks over and over the rhetorical question: What causes randomness? He finds lots of examples of (1) inherent randomness, (2) systems that perpetuate randomness, (3) and algorithms that generate apparent randomness. But he never answers the question of what it is about the third type -- the randomness generator -- that causes randomness.

In **Observer Physics** I answer the question and provide a definitive experiential model you can test for yourself. Consider, for example, the TM technique. It is a very simple algorithm. Sit down. Close the eyes. Wait a few seconds. Pick up the mantra. Continue thinking the mantra in the effortless way that you have been instructed to use it. . . . This is a very orderly and simple algorithm. What are the results? From time to time the mantra disappears, and you find yourself thinking random thoughts. TM thus appears to be a Class 4 system. It starts with a simple program, but develops into a complex localized structure of fractal versions of the mantra embedded in a background of random thoughts and experiences. Oddly enough, this is a very comfortable and natural experience -- suggesting that it is inherent to our nature. Perhaps we are "Class 4 Creatures".

Wolfram might counter: Well, the random thoughts come from "outside" the system. True enough, Wolfram has a very precisely defined algorithm and implements it in the very controlled and limited environment of a computer. But what is "outside"? The system we are using is consciousness. Does not his principle of computational equivalence proclaim that the randomness generated by a natural phenomenon, or a computer program or a human consciousness, are equivalent? The point here (discussed in detail in **OP** from an analysis of consciousness and attention) is that this "simple" process involves initial conditions that begin in a state of extreme bias (ordinary waking state consciousness, thinking the mantra). Attention is awareness flowing in a particular direction. The direction of attention initially is biased to the mantra. But then it shifts to subtler levels of appreciating the same bias. As the attention gets more and more focused in that appreciation, it actually expends less and less energy and begins to expand. The system relaxes its bias. At some point it passes the threshold of bias and relaxes into a completely unbiased state. In that condition anything is possible. Only the wave function of the body and mind of the individual generates higher probabilities for certain thought events. They function as a sort of background bias, a larger nested island complex of probability in the ocean of total thermal chaos. But the ensuing thought events occur quite randomly -- within the probability guidelines of the system. The same is true of the automaton Rule 30. Wolfram uses Rule 30 as the random number generating "mantra" for **Mathematica**. Who knows? Rule 30 might even be a good technique for meditating.

The principle of relaxation of bias at fine resolutions explains the "quantum leap" and strange phenomena such as the double slit experiment. When a system' s bias (boundary) relaxes, its "attention" becomes de-localized. When you go to sleep tonight, pay attention to the process by which your attention de-localizes. Then notice how it re-localizes when you wake up. Try deliberately de-localizing your attention. For example, imagine yourself being vastly bigger than the whole universe, such that the universe is not even noticeable. You get homogeneity. Imagine that you do not exist as an individual, and your will has completely gone to sleep. Stuff just comes and goes willy-nilly. You get randomness. Understanding how attention works explains a great deal of the mysteries of quantum mechanics.

According to Heisenberg, as the (Dx) gets to finer resolution, the (Dp) gets de-localized. When the (Dp) gets to fine resolution, the (Dx) gets de-localized. When the TM mantra gets really, really subtle, the "momentum" of consciousness expands to fill the whole universe. We experience unbounded awareness and then any thought can come up. The attention is not biased toward preferring certain thoughts.

When the momentum of a photon beam is very precisely confined, its position becomes very non-local. The photon beam goes through a tiny aperture (focused direction) and then ends up spraying photons all over the place (unfocused location). Heisenberg' s relation says

* (Dp) (Dx) >= H.

The (>) operator means that both position and momentum intervals can relax their bias and expand, which they do, if given half a chance. Thus, in the double-slit experiment, you get a single photon generating interference with itself as if it goes through both slits -- which it does -- IF you relax the bias that it MUST hold to one path. If you tighten the bias by closing either slit or by trying to measure the photon as it goes by (i.e. monitoring), then you lose the unbiased interference pattern.

The tiny aperture experiment is microscopic and invokes the Heisenberg relation. The double-slit is macroscopic and just involves the range of relaxation of the boundaries of attention focus. Attention' s conjugate partner is a photon. We only perceive photons. Everything else is imaginary. Photons are by nature non-local, but we squeeze their beams (pop qwiffs) by the focus of our attention beams. Scientists puzzle over quantum mechanics because they do not want to take responsibility for their own consciousness. However unfocused your attention is, the photon wave function will meet you at that level of focus. You can only see what you are looking for/at. All the rest is imaginary. If you have three slits, you' ll get three waves interfering. Unless, of course....

Perhaps the most important insight Wolfram is promoting in his book is something that he gained from working on his **Mathematica** project. Over the past three hundred years mathematics has undergone a process of evolution that resulted in the "liberation" of mathematics -- the realization that all mathematics is an arbitrary game designed by the mathematician. (See Hockett' s design feature of "arbitrariness".) Liberated Math is what I call "Observer Math" (OM). Anything goes as long as you are reasonably consistent and have fun. (I suppose the fun part is optional.) Goedel' s technique is a good example of using a really imaginative method to do mathematics. Wolfram has seen that this notion of liberation transfers to physics and other disciplines of science as well. He realizes that there is no "right" way to do physics, and physics then becomes an unbounded field to play in. In a nutshell Wolfram has realized (**A New Kind of Science**, p. 5) the principle of Observer Physics that the rules of science can be "rules of essentially any type whatsoever." This is the liberation of science -- something which thoughtful scientists have been aware of for some time, but have not dared to trumpet loudly because of the general population' s addiction to the "reality" of reality according to their pet beliefs.

In Chapter 4 Wolfram plays with operations on numbers as automata programs that generate various waveforms. Compare his findings with the ideas presented in the first several chapters of **OP**. In other chapters he considers (5) multidimensionality, (6) randomness as a starting point, (7) the relation between programs and nature, (8) the everyday world, (9) physics, (10) perception and analysis, (11) the idea of computation, and (12) his principle of computational equivalence. He finds that the same classes appear wherever he looks, and decides that they are universal classes.

His chapter on physics is particularly relevant to observer physics. Here are a few key points that come up.

^ He notes that principles such as conservation of energy and equivalence of directions seem unrelated to the behavior of cellular automata but can be mimicked by certain programs. This suggests that they are special cases within a much larger and more general context. The same is true of reversibility. Many automata are irreversible. Our previous discussions of pathways explain why.

^ Wolfram notices that lots of automata do not seem to follow the second law of thermodynamics. It should be clear why. He is confusing the Mind Space and the World Space. Automata belong to the Mind Space, even though they can be run on a computer. If they are precise, discrete sets of rules and operations with predictable behavior, it is possible to extend them with certainty as far as you like with no loss of information in many cases. The second law describes the World Space. Wolfram' s paper printouts are subject to the second law, but not his pure mathematical algorithms. This is a key point of **Observer Physics** discussed in chapter 1. Confusion results when we lose track of where we as observers stand vis-a-vis the crossover points in a system. Wolfram' s real insight here occurs when he turns the situation around and realizes that the second law may be a biased viewpoint that only looks at a select range of possibilities. In other words, the second law may not be as general as many people assume.

^ On page 455 Wolfram comments that the darkness of the night sky is evidence for the expansion of the universe. In fact this is only evidence supporting the belief that the universe is expanding. By shifting viewpoint one easily notices that there is background radiation everywhere. The apparent extreme red shifting of this radiation beyond our visual range may be interpreted as merely a sign of the observer' s incredible shrinking viewpoint. As an experiment go out on a clear night and observe the sky. Gently adjust your viewpoint until you can actually perceive that the sky is filled with light. Go back inside and close the door, turn off the lights, and, if you like, put your head under a blanket or go into a closet. (Alternatively you can get a sensory deprivation tank and really do some experiments.) Tune your vision until you can see the light field that persists even when all "external" lights are extinguished or blocked. Where does that light field come from? What do you believe?

^ Wolfram believes (p. 468) it will not be possible to find the Great Rule that generates the universe without already knowing it. Well, then he should already know it. Palmer' s Theorem, the Fundamental Principle of Observer Physics states it very simply and clearly with a self-referring fractal formula that automatically generates the universe, or any other universe you might like to explore. **Observer Physics** is a start at unfolding some of the consequences of that principle and showing how it links up to what we already know (strongly believe) about modern physics.

^ Wolfram believes that the nature of space-time is a huge nodal network. This is a very interesting idea and worth exploring in the light of other emerging theories such as Nottale' s fractal space-time and the notions of Observer Physics. In an aside, however, Wolfram notes (p. 476) that there are really only networks of 0-, 1-, 2-, and 3-branch nodes. The 0-branch ones are of course null networks -- that is, nodal dusts. No interactions happen with 1-branch nodes either. 2-branch systems seem trivial. All others are just 3-branch nodes or combinations of 3-branch nodes. Feynman diagrams for QED show only 3-branch nodes: 2 for electrons and one for a photon. In OP we show that the QED Feynman diagrams are really 4-branch systems. Any QED interaction is really a four-wave/four-particle mixing involving, say, two electrons and two photons. The photons have tight trajectories that are read as one. The rule about 3 branching nodes forces the interaction into a ring or bubble structure. This is an example of a phase conjugation quantum bubble. Such a bubble can be of any size. It generates the illusion of space-time. Within the bubble is a region of hyper-space-time. Interactions occur within that bubble at the Planck Velocity. An observer can choose to operate outside the bubble, or inside the bubble, or on the boundary, or as the whole system, ... A complete QED interaction involves two conjugate 4-branched nodes: for example, two electrons to two photons and then the two photons to two electrons; or a pair of photons to an electron-positron pair and back to a pair of photons.

^ It is interesting to compare Wolfram' s model of relativistic time dilation (pp. 523-524) with the klystron models we discuss in **OP**. One might also compare his automaton models of elementary particles (pp. 525-530) with the **OP** models built from decimals and with the self-regenerating dynamic lepto-quark models of **OP**.

In spite of a few minor criticisms, I enjoy this book very much. It is an excellent (probably the classic) reference work on cellular automata, and a landmark in the evolution of the New Kind of Science. Wolfram definitely deserves his "genius award." He' s doing some really good work.

Comments on Nottale' s
Fractal Space-Time in Microphysics:
Towards a Theory of Scale Relativity

Chapter 1: "General Introduction." Nottale presents his general hypothesis that addresses the problem of scaling and scale invariance and gives a brief summary of the topics to be covered in his discourse.

Chapter 2: "Relativity and Quantum Physics." Notalle gives a summary of the present state of physics, pointing out the lack of theories to predict at the smallest and largest length and time scales. He also points out the divergences (infinities) that crop up in both classical and quantum models and the inadequacy of renormalization techniques to deal with the problem. He points out the circularity of Einstein' s notion that gravitation is space-time curvature. What is the curvature there for? Is it to produce gravitation? He mentions the curious correspondence of a complex operator with a real momentum. What' s that doing there? Where is it? What is quantum foam? The Riemannian geometry of relativity fails to handle the quantum world. New concepts are needed to extend "Relativity". He defines the three forms of relativity involved in "locating" an event: (1) a **ratio** to axes with an origin; (2) a **unit** of measurement; and (3) a **resolution** that indicates the "precision" of a measuring method (i.e. a limitation of some kind). [In Observer Physics we discuss these three relativities as the **ratio**, the **unit**, and the **scale** of a quanstant or other physical measurement. For example,

* c = 3x10^8 m/s.

Here (3) is the ratio, (m/s) is the defined unit, and (10^8) is the scale. These three relativities are all set by the observer.] Units are necessary for any measurement because all measurement is a comparison of two relative phenomena. Notalle opts to retain the space-time continuum hypothesis while adding the notion of fractal structure. He also points out that in quantum physics there appears to be limiting scale that is "invariant under dilatation" (p. 26) and corresponds to the role of (c) for the laws of motion. Like (c) it is not a "cut-off", or a "quantization" or a "discontinuity". We must give up Minkowskian space-time and find a new space-time model that fits both relativity and quantum mechanics. Notalle posits a "zoom dimension" and "zoom space-time" using (Dt, Dx, Dy, Dz) alongside the usual (t, x, y, z). [These (Dx)' s and so on correspond to the "peanut" or "gap" numbers that we introduce in **Observer Physics** and provide the resolution at which an event is viewed by an observer, while at the same time ensuring continuity.] The observer defines the exact size of a (Dx). Notalle uses little "balls" to smooth out the fractal curves into approximations with finite lengths with measurable slopes. The fractals are inherently infinite in length and have undefined slopes -- which render them non-differentiable. [This "ball technique" is similar to the Snow White and Seven Dwarves quantum foam approach we adopt in chapter 13.]

Chapter 3: "From Fractal Objects to Fractal Spaces". In this chapter Notalle introduces in detail the concept of fractals, providing examples of fractal objects that have topological "dimension (DT) and fractal dimension (D), such that D>DT." (p. 33)

The discovery that fractal structures occur commonly in nature brings up the desire to explain why they occur in nature. Notalle suggests that fractal geometry is a generalization of the ordinary Euclidean and non-euclidean geometries that unfold in whole number dimensions. If it turns out that fractal structures dominate over non-fractal structures in nature, then the use of differential calculus may require essential revision. Fractals exhibit the paradoxical condition of divergence combined with boundedness. Self-similarity allows a fractal to remain identical locally after undergoing dilation or contraction. Notalle refers to this as "a periodic system in the ' zoom' dimension." (p. 40) He also notices that threnormalization group approach in QED, for example, starts from the infinitesimal and works upward with patterns of self-similarity, whereas fractals are usually generated from a macroscopic generator that is then iterated at increasingly smaller scales. He suggests that these might be conjugate forms -- that is, inverse transformations. [In observer physics we discuss entropy (kinetic energy) and gravity as inverse transformations.] Notalle develops the mathematical description of fractals and points out that fractals are usually envisioned as embedded in Euclidean space. He proposes going beyond that idea to a fractal space-time and giving up the notion that the curvature of space-time approaches Euclidean space as a microscale limit. In fractal space a trajectory' s curvature approaches infinity as (Dx) approaches zero. Of particular interest in this chapter is Notalle' s discussion (section 3.4) of NSA and the Robinson hyper-reals (supersets *R of R that include infinites and infinitesimals) as tools for developing notions of a fractal derivative and fractal integrals. This leads to reformulating the Cauchy-Weierstrass limit used to define the calculus. (See **Observer Physics**, chapter 1.) Notalle shows a sample fractal curve representing the number 0.11111.... (p. 60) [From this example it is clear that we can reinterpret all the infinite decimals that we play with in **Observer Physics** as fractals. Numbers such as 0.1111... take on unique interpretations. The number 1.000... is definitely not the same as 0.111.. if we view it fractally.] Using NSA techniques Notalle can extract finite solutions (approximations) from non-standard differentiation of fractal curves. He also considers fractal functions, variable fractal dimensions (e.g. curves that tend from D = 1 to D = 2 as they go along or curves that vary their level of resolution) and leads us toward a definition of fractal space-time.

Chapter 4: "Fractal Dimension of a Quantum Path". In this chapter Notalle begins to apply his ideas to the foundations of physics. He begins with Feynman' s discovery that quantum trajectories are fractal. Each quantum path is equiprobable, but its "probability amplitude" has only a phase term. So from the view of "infinite precision" all paths are equiprobable, but when viewed from an arbitrary resolution, paths divergent from the classical path cancel out. Paths very close to the classical path are wiggly and fractal. The transition from quantum fractal behavior to classical non-fractal behavior occurs, Notalle proposes, at the de Broglie scale. (p. 91) Quantum systems are characterized, he believes, by D = 2. Classical systems exhibit D = 1. The jump occurs at the de Broglie length:

* Ldb = H / p.

Notalle sees this as a quantum relativistic path. He plots a doubly relativistic (scale and motion) diagram (p. 100) to show two transitions -- a spatial coordinate transition at the de Broglie length as (Dx) increases, and a temporal coordinate transition at the de Broglie time as (Dt) increases. So the quantum relativistic dimensions are D = 2 for both space and time. In the quantum domain time becomes D = 1. In the classical domain, both space and time are D = 1. Notalle refers to the fundamental phenomenon that in the relativistic domain particle-pair creation-annihilation occurs. Measuring at the precision of

* Dx = H / M c

in the particle' s rest frame puts the uncertainty on the threshold of pair creation. The limit for position is

* Dx = H c / E.

In observer physics we identify the ratio (%) as the ratio of (H c) and place that as the spatial interval for the minimal pair-creation threshold:

* Mne = H / c %.

At this energy level, light waves can begin to manifest as neutrinos. Nottale stresses the loss of precision with regard to position (Dx) when we are dealing with quantum particles. This is particularly obvious with neutrinos, the lightest particles.

Notalle suggests that non-relativistic quantum time may take the form of a fractal dust with a fractal dimension of 1/2 and a topological dimension of 0.

The jittery quantum trajectories (Zitterbewegung) of quantum particles indicate folding in time and virtual pair production. He interprets these as fractal space-time. Fractal space-time defines quantum particles in the same way that curvature defines gravitation in Einstein space-time.

Chapter 5: "The Fractal Structure of Quantum Space-Time."

In this chapter Nottale goes on to develop his idea that fractal space-time defines quantum particles in the same way that for Einstein space-time curvature defines gravitation. The particle becomes its own trajectory and is thus just a manifestation of fractal geometry.

Nottale mentions a Peano curve that, depending on the angle one views it from, either has a "constant" slope of 0 or a periodic slope that flips back and forth between 1 and -1. He suggests this as a model for shifting from quantum interference to classical wave structure. He develops a fractal picture of how a measurement may affect a fractal trajectory causing unpredictable results. He shows how uniformity appears only at the ideal fractal level and disappears when approximations are made at various resolutions. He also develops the way in which a fractal trajectory may resemble a probabilistic distribution.

He discusses the problem with velocity and the speed of light in microphysics and shows how it varies with regard to the scale of resolution. Then he brings up the problem of electron quantum spin. The speed at the surface would seem to exceed light speed. He suggests that certain types of fractal may satisfy the observed spin phenomena with quantum particles. Another idea he presents is that Goedel' s theorem applies not only to mathematics, but to physics. Quantum physics, he believes, is encountering the same occurrence of non-demonstrable, but true, situations that mathematics does.

When he discusses the deeper issues of the EPR problem, randomness, stochastic quantum mechanics, and the role of Brownian motion, there is a sense that he is begging the question. We do not know where the randomness comes from. However he does make the cogent point that relativity of motion should be extendable to include non-differentiable states and thus include fractal dimensions. He would like quantum mechanics to achieve Einstein' s goal of derivation from "fundamental principles" (p. 136), but, like Milgrom' s MOND formula that lacks a principle, Nottale sees that the current QM stochastic processes still fall far short of that mark (p. 137). Nottale attempts to go further with his postulate of a fundamentally fractal space-time, but admits that he still falls short of Einstein' s ideal (p. 143).

Nottale discusses at length in 5.7 the transition between quantum and classical dynamics. He believes that the transition is reversible and defined by the thermal de Broglie wavelength (Lt):

* Lt = H / (2 Mx k T)^1/2.

Here (Mx) is the particle' s mass, (k) is Boltzmann' s constant, and (T) is the absolute temperature (Kelvin). Oddly enough, a Brownian motion can push a system from quantum to classical or vice versa. Although this "explains" how a motionless (Vx = 0) macroscopic object can have precise position, but infinite de Broglie length (H / Mx Vx), it does not explain where the thermal (or Brownian) energy comes from. Nevertheless, Nottale claims that the fractal approach provides "information about the virtual internal structure" (p. 159) of a particle' s trajectory.

In section 5.9 Nottale shows how QED Feynman diagrams are really fractal structures and in 5.10 he discusses the anomalous heavy ion collisions as possibly following fractal patterns. He suggests that current models of quantum mechanics may require revision under new findings from strong field experiments (such as the anomalous Darmstadt heavy ion collision findings) and at the very small (high-energy) scale.

In 5.11 he begins his discussion of General Relativity to develop his theory of scale relativity. He notes the problem that when a mass gets beyond the Planck mass, its Compton length gets below the Schwarzschild radius. We can not measure such a length because it is inside a black hole. On the other hand the Schwarzschild radius lacks physical meaning below the Planck mass. This leads him to his major point that the Planck length is an unpassable lower limit somewhat like the speed of light (c), and thus forms a universal constant. On p. 188 he has a chart of the Compton length

* Lc = (H / Mx c)

and the Schwarzschild radius

* r = (2 G Mx / c^2)

plotted in terms of (ln Mx) versus (ln r). He proposes that as the two relations approach the Planck length, they curve asymptotically. In **Observer Physics** we not only propose the Planck length as a minimal scale, we show how it fractally generates our human scale lengths of Ru = 1 meter and % = 3.1622 meters via the Heisenberg relation. This defines the relation between the Joule and the D-Shift Gauge and the Planck scale.

* [(P %^2 / Ao)^-26 (J / c)] (%) = H.
* H c / [(P %^2 / Ao)^-26 (%) = 1 J.

The D-Shift Gauge is a major tool for shifting scale in quantum mechanics. It will be very helpful to Notalle in refining his theory.

In section 5.11 Nottale goes right in for a close look at the Planck mass (p. 187-190), and then shows how relativity and quantum mechanics both need some fixing. He does what we do in **Observer Physics** -- he looks at the Planck mass, Compton length, and Schwarzschild radius. What he sees is a critical point, a barrier where physics breaks down. In **Observer Physics** we see this situation as the window of opportunity to a new physics. Notalle, from his fractal angle, has the same hunch. He then brings up what is basically the Millikan experiment -- he notes that "the gravitational force between two Planck masses is the same as the Coulomb force between two bodies each of them carrying (about) 12 elementary electric charges." The "12" here is actually due to the fact that Nottale left out the fine structure constant when he compared the two because he was figuring from the traditional value of the Planck mass. But he definitely got close to seeing the picture.

* Mpl = (H c / G)^1/2.

Having come from the angle of first calculating the equilibrium of gravity and the Coulomb force, we arrive at a figure that differs only by the fine structure constant, but neatly unifies these two viewpoints of gravity and electricity and connecting it to the Planck mass.

* Bu = (H c a / G)^1/2.

When we take the square root of (a) = 137^-1, we get approximately (1 / 12). This gives us a particle size that exactly balances a **single quantum of electric charge**. This becomes clear when you calculate the balance point between gravity and the Coulomb force and look at it together with the Planck mass, Compton length, and Schwarzschild radius calculation. Here' s the whole story in a nutshell (See **Observer Physics**, Chapter 12.) leading to the relation between quantum charge, the Planck scale, and light speed.

* Mpl = (H c / G)^1/2 (Mpl = Planck mass)
* Rs = 2 G Mbh / c^2. (Rs = Schwarzschild radius of black hole)
* Lc = H / Mbh c. (Lc = Compton length for a quantum black hole.)
* Mbh = (H c / 2 G)^1/2 (Mbh = Minimal black hole mass)
* Fe / Fg = e^2 / 4 P eo G Mp Me (Fe / Fg = Ratio of Coulomb force to Gravity)
* 1 = e^2 / 4 P eo G Mx^2. (Unification of Coulomb and Gravity forces)
* Bu = (e^2 / 4 P eo G)^1/2 (Bu = Union Particle Defined)
* Bu = (H c a / G)^1/2 (Bu = Equivalent in terms of Planck Mass)
* Mpl = (2)^1/2 Mbh = (a)^-1/2 Bu (Relationships of Mpl, Mbh, and Bu)
* (e^2 / 4 P eo G) = (H c a / G) (Combining the two equivalent expressions)
* e = [(4 P eo) (H c a)]^1/2. (e = Derivation of quantum charge unit)
* a = (e^2 / 4 P eo H c) (a = fine structure constant definition)
* G Mx^2 = H c a = e^2 / 4 P eo. (The fundamental relationship)
* 4 P eo = (10 / 9)(10^-10) kg / m^3. (Mass-energy density constant.)
* 4 P eo = [(Ss P % / As Ao)^2 (P %^2 / Ao)^-(P %^2 / Ao) kg / m^3.
* (H c) = (%) [(P %^2 / Ao)^-26 J]. Planck-scale D-Shift ("Sweep ") constant

The fine structure constant (a) comes up as the coupling constant when the Planck mass self-interacts to form a pair of Union Bosons. This pair finds equilibrium as the proton ensemble and becomes a persistent mass. Each time the proton iterates, (a) gets squared. If it can' t find equilibrium as a Boson pair, the Planck mass is a virtual particle that self-annihilates by Hawking radiation. These entities clearly have fractal or quasi-fractal structures. Here Nottale again is definitely on the right track and gets very close to seeing how it works. He even suggests some experiments that could be done studying the gravitational interactions of dust grains. Such experiments are quite recommendable. (See diagrams of the Planck mass fractal structure at end of this review.)

Chapter 6: "Towards a Special Theory of Scale Relativity"

Having established his notion of an "unpassable lower scale to Nature" as a natural unit for scales, Nottale begins developing his theory of scale relativity. His absolute and universal scale is independent of any particular physical object and based only on the three physical constants (H), (c), and (G).

* Lpl = (H G / c^3)^1/2.

One principle that immediately emerges is that the scale relativity is not an ideal fractal form, because it has a lower limit at the Planck length and it breaks at the de Broglie transition. Another principle that emerges is that laws of scale replace laws of motion under high-energy conditions (p. 196). This is similar to the way that time and space switch roles on either side of (c).

Nottale first reviews the successes and tricks (e.g. renormalization) used in QED and the

GUT theories. Then he points out many discrepancies that have emerged and the *ad hoc* nature of many fixes. He points out that both measurements and numbers require scales, and we should find the most fundamental and universal scale as our master scale. He proposes the Planck scale. Relativity finds a dimensionless value comparing a relative scale to an absolute scale (v / c) and (1 - (v^2 / c^2))^1/2, based on Einstein' s principle of the limit (c). This seems to be the direction Nottale is moving to find solutions to physics problems based on his scale relativity principle with its fixed Planck length.

Nottale discovers in 6.5 that his laws of scale transformation follow a Lorentzian form, not a Galilean form. In 6.9 he develops "Generalized de Broglie and Compton Relations", "Generalized Heisenberg relations", and a "Transformation of Probabilities". Then in 6.10 he looks at the "Implications for High Energy Physics", considering the divergences of mass and charge and making some predictions.

On p. 258 he is drawn back to marvel that, within a factor of 2 (by his calculations):

* Lpl = H / Mpl c = G Mpl / c^2.
* Mpl = (H c / G)^1/2.

As mentioned above, in the beginning of Chapter 12 of **Observer Physics** we discuss this remarkable situation in some detail.

In 6.11 Nottale derives some interesting formulas for the masses of the intermediate vector bosons in which he connects them to the Planck scale and to the electron scale.

Toward the end of Chapter 6 (p. 276) Nottale again gets hot on the trail with his notion that charge is what he calls "fundamental dilatation". In **Observer Physics** (Chapter 10) we discover what this "fundamental dilatation" really consists of and how it is generated when we discuss the question of electric charge.

Chapter 7. "Prospects."

In this final chapter Notalle opens up his principle of scale relativity to look at cosmology and the large-scale vision of the universe. This chapter is far ranging and much more speculative than the earlier chapters. He begins by considering the curious ability to express the electron mass in terms of the Hubble constant (Ho ~~ 3x10*-18 s*-1) that many have noticed.

* (H^2 Ho / G c)^1/3 ~~ Me a_.

But what do you do with that? The Hubble "constant" changes over time. Does the electron mass then change too? Or (H), or (G), or (c), or (a_)?

Nottale plays with a revival of Mach' s Principle. He notes that this principle makes the universe into a black hole. Following Sciama he mentions the idea of gravitational induction on the analogy of electromagnetism. He considers masses compared to the mass of the universe and compared to the Planck mass. For example, using his Planck scale principle, Nottale suggests that Newton's second law could be rewritten as:

* F = H c [(Mx) / Mpl) (My / Mpl) / r^2].

Nottale starts the Big Bang clock at the Planck time, not at zero time. He notes that causally disconnected parts of the universe should act independently, but strangely the background radiation is quite even. He feels the inflationary theories are *ad hoc*. He proposes that scale relativity changes the behavior of light cones, causing them to flare wider in the primeval universe. Thus he resolves the strangeness issue without resorting to (or perhaps we should say in a manner that looks equivalent to) inflation. This means that at the resolution of Planck time all points of the universe are causally connected. (See his excellent chart on p. 293.) **Observer Physics** agrees with this conjecture for reasons that involve the role of the Observer, a factor that Nottale, for all his creative ideas, insists on ignoring.

Further on (p. 296) he brings up Laplace's wonderful remark that Newton's law of gravity, when scaled, reproduces the universe at any scale. That is -- it is scale invariant. Nottale goes on to mention the dispersion relation for objects in a gravity field.

* *l* = G Mx / Vx^2.

He takes this as a macroscale version of the microscale de Broglie length:

* Ldb = H / Mx Vx.

To get these to be equal, you discover:

* Mx = (H Vx / G)^1/2 = Mpl (Vx / c)^1/2.

He points out that, if Vx = c, then (Mx) becomes the Planck Mass, (Mpl). This connects the two domains at the microphysical level (p. 297.)

He looks at the vacuum energy density (#pl). In standard QM this diverges. By setting the Planck scale limit Nottale gets:

* #pl = (c^5 / H G^2).

Later (p. 301) Nottale notes that the "Universe at its own resolution" is invariant under dilation. He suggests the Einstein spherical model as this viewpoint, with local variations. This leads him to his ratio of the mass of the universe (Mu) to the Planck mass.

* Mu / Mpl = P Ku / 2.

Here he estimates the value of (Mu) to be 10^53 kg, or 10^23 solar masses, or 10^11 galaxies with an average of 10^12 solar masses each. Nottale suggests carrying Mach's

Principle to the point of connecting the masses of elementary particles to the mass of the universe after the fashion of the Hubble constant coincidence. By analogy with his lower limit Planck length (Lpl) he posits an upper limit (Lu). You can't get beyond it, even with the universe expanding. This length scale stands in for infinity. He suggests a ratio (Ku) between these two lengths (p. 299).

* Ku = Lu / Lpl.

He relates this to the cosmological constant (Lcc).

* (Lu)(Lcc)^1/2 = 1.

From this he estimates the value of (Ku) to be around 10^61.

He identifies the smallest possible energy as

* Emin = H c / Lu.

* Mpl = (H c / G)^1/2 = 2.176x10^-8 kg.
* a Mpl / Me = Ku^1/3 = 1.7437x10^20.
* (Ku) = 5.3018x10*60

Nottale again comes in really close with his conjecture. He defines a scale (ro)

* (ro) = a Lc = (a H / Me c).

This is the Lorentz electron radius, with (a) equaling the fine structure constant and (Lc) as the Compton length for the electron.

He considers the electron as purely electromagnetic, putting it in terms of (e). In our notation we would write Nottale's expression:

* e^2 / 4 P eo = (Me c^2) (ro).

Substituting the value of (ro) and then the value of (a) in constants, we get:

* e^2 / 4 P eo = (Me c^2) (a H / Me c)
* e^2 / 4 P eo = H c a.
* a = e^2 / 4 P eo H c.

Then Nottale puts the electron's "gravitational self-energy" at the (ro) scale at his minimal energy (H c / Lu).

* G Me^2 (ro) / (ro) = (G Me^3 / a^3) (c / H) = H c / Lu.
* (Me / a)^3 = H^2 / G Lu.
* (Me / a)^3 = H^2 / G Lpl Ku.

* Lpl = (H G / c^3)^1/2.
* Ku = (a / Me)^3 [H^ (4/2) G^ (-2/2) (H G / c^3)^-1/2].
* Ku = (a / Me)^3 H^3/2 c^3/2 G^-3/2 = (a Mpl / Me)^3.
* (a Mpl / Me) = Ku^1/3.

What is remarkable here is how close Nottale comes to the simple figures for the Union Boson. He does not have to make all those assumptions. Or, conversely, we can say that the calculations of **Observer Physics** provide a simple way to support his assumptions as basically correct.

* Bu = (H c a / G)^1/2.

We simply reorganize the details of his calculation and the Union Boson magically appears -- almost.

* Ku = (a / Me)^3 [H^ (4/2) G^ (-2/2) (H G / c^3)^-1/2].
* Ku = (H c / G)^3/2 (a / Me)^3.
* Ku = (H c a / G)^3/2 (a / Me^2)^3/2.
* [(H c a / G)^3/2] (a^3/2) / Me^3 = (a^3/2) (Bu^3 / Me^3) = Ku.
* (a^1/2) (Bu / Me) = Ku^1/3.

Here again is that factor of (a^1/2) or about (1/12) that Nottale was off by in his other calculation where he looked at the Planck mass, the electrical force and gravity. So we would suggest that the proper equation simply may turn out to be:

* Bu / Me = Ku^1/3.

This boosts the value of (Ku) up to around 8.5x10^63.

The closeness of Nottale's findings to our findings in **Observer Physics** suggests that there is much interesting research to do in this area.

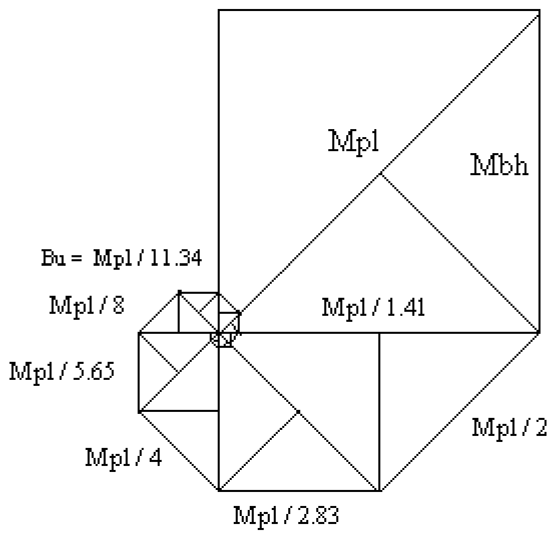
Nottale notes the problem of global uniformity not squaring with his fractal idea at the cosmic scale and admits the tentative nature of his conjectures.

In 7.2 he goes "Beyond Chaos", considering some other speculative notions such as "prediction beyond predictability", an interesting fractal model of the solar system, and the use of fractal models for studying chaotic systems.

The strong evidence uncovered in **Observer Physics** of ratios that echo in a fractal-like manner at various scales throughout the relationships of the physical constants suggests that Nottale's theory is moving in the right direction. More emphasis on the critical role of the observer in physical events may be helpful.

In summary, I consider that Nottale's book is well worth careful study.

The Fractal Structure of the Planck Mass



The above diagram is not exact. As the nautiloid fractal spiral goes to smaller scales, there is a slight warp to it that you can' t see in this flat projection. The average scaling factor comes out to a tad over 1.421, which is just slightly above (2)^1/2. This gives the following approximate scaling dimensions:

* **Mpl**, **Mpl / 1.421 = Mbh**, Mpl / 2.019241, Mpl / 2.86934, Mpl / 4.077, Mpl / 5.79, Mpl / 8.233, **Mpl / 11.7 = Bu**.

* Mpl = (H c / G)^1/2.
* Mbh ~~ Mpl./ (1.421) ~~ Mpl / (2.019241)^1/2.
* Bu ~~ Mpl / (11.7) ~~ Mpl / (1.421) ^7 = Mpl / (2.019241)^7/2.

We have "squared" off our minimal black hole (Mbh). The minimal Black Hole (Mbh) forms the radius of the "hole". Four Planck masses distribute around the perimeter, generating the archetype of "four-particle mixing." The mass ratio of the two is approximately (2)^1/2, but closer to 1.421.

^ Mpl / Mbh = (2)^1/2 ~~ 1.4.

The eccentricity (Ke) of the Bu Boson pair is 11.7^-1 = .0854... This is the **square root** of the fine structure coupling constant (a)^1/2. The relationship to the Planck mass is shown in the fractal diagram.

* Bu^2 = H c a / G = H c Ke^2 / G.
* Ke = 11.7^-1 = .0854...

In general, for gravitational systems of any size:
.
* (M1) (M2) = Fg D^2 Ke^2 / G.

(Fg) is the gravitational force, (D) is the distance from an orbiting object to the nearest point on its directrix, and (Ke) is the eccentricity of the orbit. We find the eccentricity for the Boson pair as shown above. For details see **OP**, chapter 13.

The fractal dimension (a)^1/2 tells us the eccentricity, or "bending", of space-time that occurs when the Planck mass forms a pair of Bu bosons that interact and reach equilibrium. This bending gives space-time a fractal structure and simultaneously generates the appearance of the gravitational, electric, and magnetic forces according to the fractal relationships shown below.

* c^2 = 1 / eo mo.
* (a) = e^2 / 4 P eo H c = Ru^2 e^2 / As eo H c.
* c^2 = 1 / eo mo.
* (a) = (Ao As / P % Ss)^6 (Ao / P %^2)^2 = (As^2 Ao / Ss^2 P)^3 (Ao / P %^2)^5.
  (137)^-1 = (1.054)^-6 (10)^-2 = (729) (10)^-5

* e^2 / eo H c = (Ao As / P % Ss)^7 (Ss / %^3) = .0917. (7 ~~ P^6 a.)
  (1.054)^-7 (.132461)

In the schematic diagram you can see how the minimal Mbh black holes come together and interact, warping space-time into a crinkled space within which a Boson particle appears. The Planck mass' s space-time then gets all crinkly like a fractal raisin, since the Bu mass is smaller than the Planck mass. The crinkliness manifests as electric charge and magnetism, the two forces being oriented orthogonal to each other.

The Planck mass is about 1.4 times the mass of the theoretical minimal black hole and 11.7 times the mass of a Bu Boson. But what we really experience is the Bu Boson when it forms bubble-pairs and achieves a dynamic equilibrium. This becomes the proton ensemble with its retinue of leptons and quarks. The density of the Planck mass at the Planck length cubed is huge (5x10^97 kg / m^3). But scaling and Heisenberg uncertainty spread it out. The diagram above only shows the mass represented as a relative length ratio. When the mass-energy actually forms a neutrino or a proton ensemble, the energy is rarified and spread out in space-time, comparatively speaking.

The key fractal ratios as seen from our way of doing physics are (1.054), (3.16227766), and (3). They form the natural relationship:

* (3.16227766 / 3)^1 = (1.054)^1.

These three ratios occur in physics as

* (%) = 3.16227766.. m = D-Shift Operator,
* (c) = 3x10^8 m / s = Velocity of Light in Space.
* (H) = 1.054x10^-34 kg m^2 / s = Planck' s constant.
* (H c) / % = 10^-26 J.

Squaring the "magic" ratio generates a dimensional shift
* (3.16227766 / 3)^2 = (1.054)^2 = (10 / 9) = 1.1111....
* 0.000, 1.000, 2.000, 3.000, ... to
* 0.000, 1.111, 2.222, 3.333, ...

Each dimensional shift is mediated by the coupling constant (a)

* (3.16227766 / 3)^0 = (1.054)^0 = 1
* (3.16227766 / 3)^1 = (1.054)^1 = 1.054
* (3.16227766 / 3)^2 = (1.054)^2 = (10 / 9) = 1.1111....
* (3.16227766 / 3)^3 = (1.054)^3 = 1.17.... = (a)^-1/2 / 10 = (10 Ke)^-1.
* (3.16227766 / 3)^4 = (1.054)^4 = (100 / 81) = 1.234567.... (number D-shift)
* (3.16227766 / 3)^5 = (1.054)^5 = 1.3....
* (3.16227766 / 3)^6 = (1.054)^6 = 1.37 = a^-1 / 100.
* (3.16227766 / 3)^7 = (1.054)^7 = 1.4455
* (3.16227766 / 3)^8 = (1.054)^8 = (10000 / 6561) = 1.524 = (1.37)(10 / 9).
* (3.16227766 / 3)^9 = (1.054)^9 = 1.6
* (3.16227766 / 3)^10 = (1.054)^10 = 1.69..
* (3.16227766 / 3)^11 = (1.054)^11 = 1.78....
* (3.16227766 / 3)^12 = (1.054)^12 = 1.37^2 = a^-2 / 10000 = 1.877

This fractal structure scales by the three values involved in the relationship (10, 9, 1.11..), with the fine structure constant mediating.

The geometry builds structures from the ratios of P, 2, and 3.

* Oo = 2 P Ru.
* Ao = P Ru^2.
* As = 4 P Ru^2.
* Ss = (4 / 3) P Ru^3.

The physical ratios fall very close to these values.

* Mp = 1.67 ~~ (5 / 3)
* G = 6.672 ~~ (20 / 3)
* e = 1.602 ~~ (4)*2.
* c = 2.9979 ~~ (3)

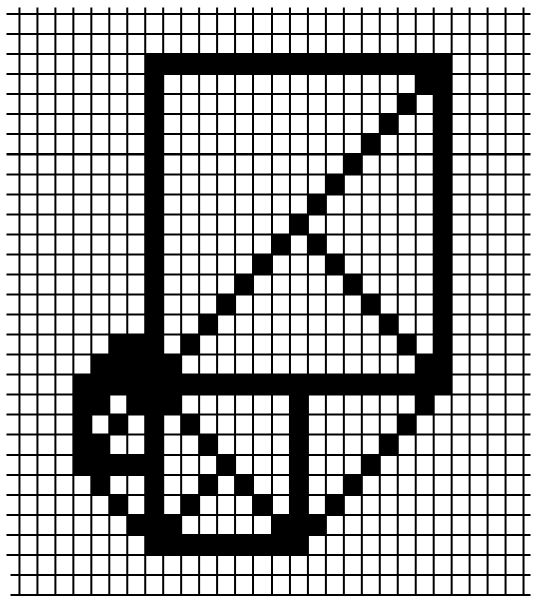
And the nautiloid spiral we constructed is, of course based approximately on 2.

Recently I went into our local Blockbuster Video store to rent a movie and discovered that they were selling off all the old VCD' s at highly discounted prices and replacing them with DVD' s. On the way home I passed several shops filled with young people playing video games. When I reached home, I turned on my computer to continue writing this discourse.

Our world is rapidly shifting from an analog paradigm to a digital paradigm. Yet many still cling to the old ideas about continuity. And, from a certain perspective, it is a valid viewpoint. But I think it is time to start learning to think digitally from a basic level.

Notalle claims that he is working from a continuum hypothesis, and yet he also declares that the Planck length forms an unpassable lower limit on resolution. This suggests that every object that has size is built from quanta of the Planck length, and therefore is fundamentally digital in nature.

We treated the nautiloid binary fractal spiral that we described above as a continuous structure and applied the traditional Pythagorean relation to "measure" it. What happens if we draw this (as we actually did on the computer) as a digital figure?



The first thing we notice is that the spiral shrinks to a unit square (presumably of Planck length) and then stops. As we move around the spiral, at each rotation of 45 degrees we get either a bigger square or a smaller square. The size shift appears to be governed by the factor (2)^1/2. However, if the figure is digital the whole mathematics changes. What happens to the Pythagorean relation?

* A^2 + B^2 = C^2.

Let' s say that A is 9 squares long, and B is 9 squares long. We count the number of squares in C, and discover that C is also 9 squares long! Apparently, if the triangle is a right isosceles figure, then

* A = B = C.

Otherwise C has the same number of squares as A or B, whichever is largest.

These rules hold when sides A and B are oriented in phase with the square tiles -- the "smooth" orientation. When sides A and B are turned 45 degrees to the "jaggy" orientation, then the rule is different.

Here is the pattern for jaggy isosceles right triangles:

*	A and B	C
	1	1
	2	3
	3	5
	4	7
	......	

* C = A + 2 or B + 2.

How does a nautiloid spiral manage to expand or shrink? Let' s follow a simple example in the shrinking direction.

*	Smooth Side	Jagged Side	Smooth Diagonal	Jagged Diagonal
	17 -->			17 --> 9
		9 -->	17 --> 9	
	9 -->			9 --> 5
		5 -->	9 --> 5	
	5 -->			5 --> 3
		3 -->	5 --> 3	
	3 -->			3 --> 2
		2	3 --> 2	
	2 -->			2 --> 1
		1 -->	2 --> 1	
	1 -->			1

We use an odd number of rows and columns for better axial symmetry. A square figure that is rotated 45 degrees out of phase with the tile grid is ambiguous as to the actual length of its edge. In one sense the edge and the diagonal are equal, and in another sense the edge is one half the "size". It is very curious that, when the edge is jagged, half a square' s diagonal can be equal to an entire edge of the square. Draw them and count the tiles to get a good feel for digital behavior.

If that' s not enough, there is yet another way to look at the situation. What if we count all the runs and rises? Then we discover for our isosceles right triangle that

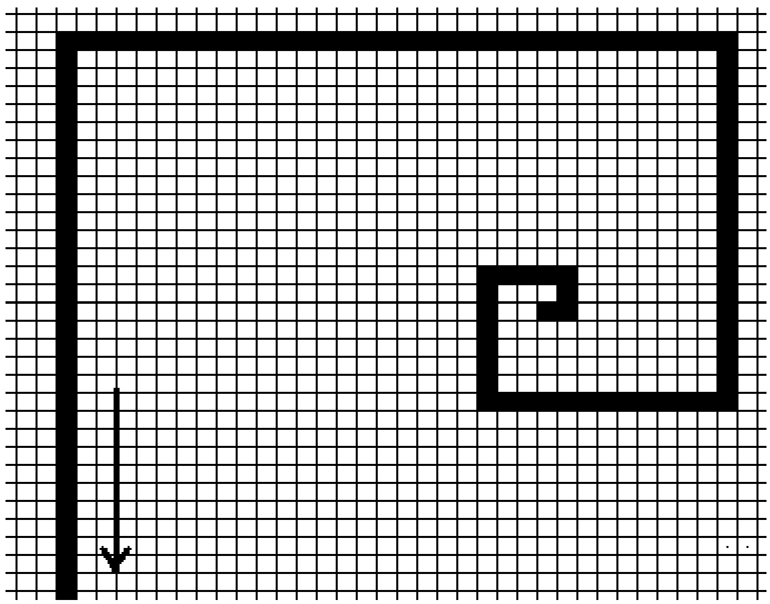
* C = 2A = 2B.

And if all sides are unequal, then the length of the hypotenuse C may vary anywhere from the longest side plus two up to twice the longest side when A equals B.

Edges can be smooth or jagged. The above analyses hold for smooth edges on the sides of a right triangle. If sides A and B are each "jaggy" 3, then the C is 5.

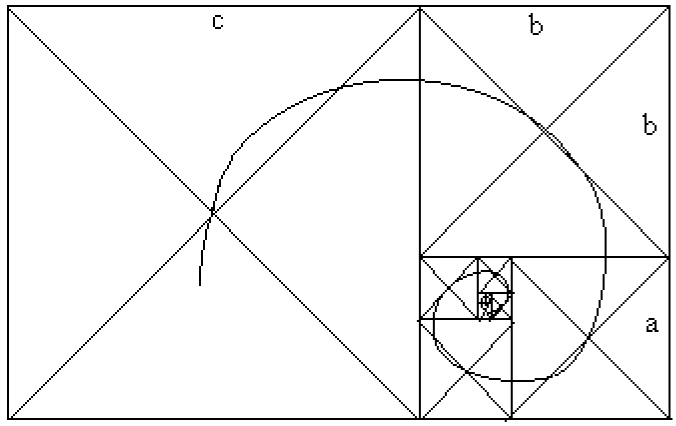
Exercise: See if you can generalize the rules for triangles made with digital squares viewed by the square or by the runs and rises. What happens to pi when you draw circles? If you are more ambitious, take a look at hexagonal tiling or other forms of tiling.

Whatever the tiling, if we coarsen the level of resolution (that is, shrink the individual tile size compared to the figures drawn), then any digital system approaches Euclidean geometry as a limit. Another interesting point is that the rules vary according the orientation of a figure with respect to the tiling background.



It is interesting to study another pair of mathematical structures that have a curious relationship and resemble the nautiloid figure we just discussed -- the fibonacci series and the phi spiral. These both have the same fractal structures, but the phi spiral is continuous and can expand or contract indefinitely. On the other hand, the fibonacci sequence, however, is digital, and can only expand indefinitely. It begins with 1. Yet the two sequences mapped as spirals rapidly converge as they expand and approach each other' s paths as limits.

* [$u1 = u2 = 1$ and $u_{(k+2)} = u_{(k)} + u_{(k+1)}$]
* 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, ...



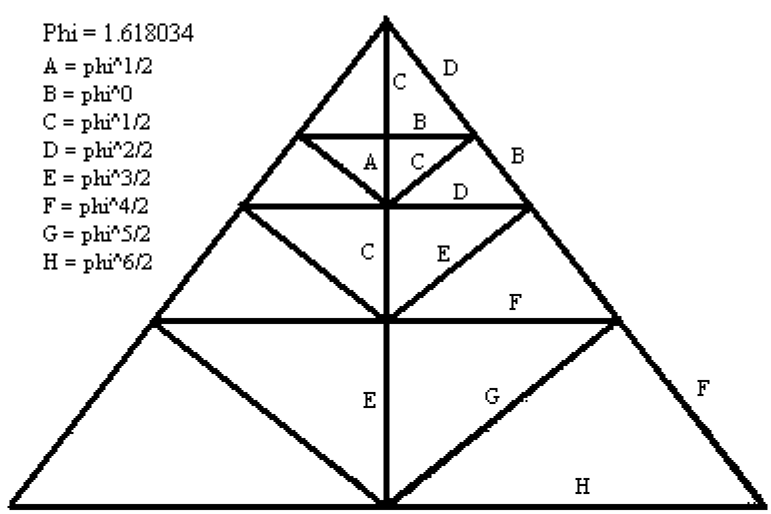
Phi = (a + b) / a = (b + c) / b ... = 1.618...

The phi spiral derives from the golden section. It has no theoretical limit in either expansion or contraction. The fibonacci spiral rapidly converges on the phi spiral.

.....
* .618 + 1 = 1.618
* 1 + 1.618 = 2.618
* 1.618 + 2.618 = 4.236
* 2.618 + 4.236 = 6.854

.....

The Great Phi/Pi Pyramid as a Klystron



The Great Phi Pyramid of Giza

The Great Pyramid at Giza is constructed so that the ratio of half the base perimeter to the altitude equals pi (P). If we set the perimeter at 8 units, half the base on one side is 1 unit, and half the base perimeter is 4. The apothem (a perpendicular from apex to base along a side) equals phi, so the altitude is phi^1/2, which also equals (4 / P), or 1.27.

* (1 + phi)^1/2 = phi.
* 4 (4 / P)^-1 = P.

Thus the pyramid is a kind of space-warped circle, since the ratio of half the perimeter of a circle to its radius is P.

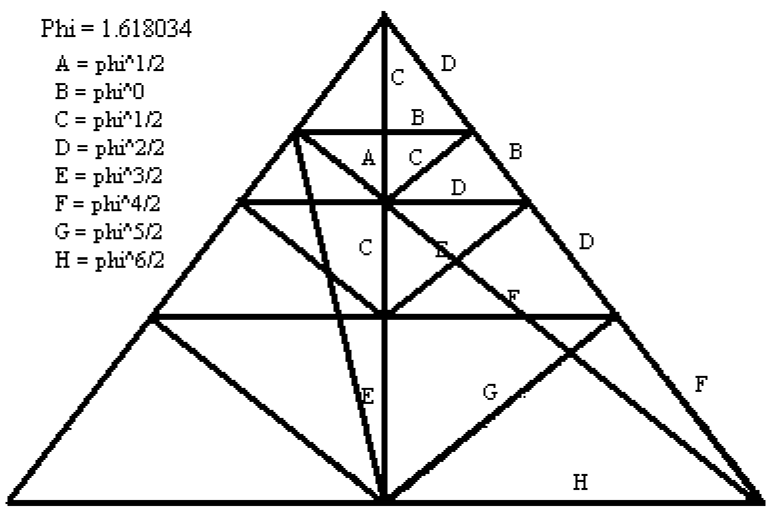
Inside the pyramid we see a cascade of triangles all sides of which are related by half powers of phi. Phi^0/2 is unity. If we take the base with perimeter of 8 units as our "capstone" and extend it downward, the phi cascade grows as shown in the drawing above. A curious feature is that the third triangle down (BCD) is equal to the top triangle (BCD). The pyramid is thus holographic. All the infinite tiny triangles that lead from the tip to fill out (BCD) are recapitulated in the echoed triangle (BCD). This tells us that there is also another virtual pyramid that extends outward at 90 degrees from the capstone.

The magical phi ratio happens to produce the same relationship as the Einstein/de Broglie Velocity relation. This means that the pyramid can function as a fractal klystron wave guide for electromagnetic radiation.

* (Vg)(Vp) = c^2.
* c t / Vg t = Vp t / c t.
* [(phi)^1/2] / 1 = phi / phi^1/2.

You can take the sketch of the klystron on page 6-4 and place it by the pyramid drawing on the previous page. Turn the klystron drawing upside down and compare it to the zigzag BCD that forms between the altitude and the apothem. Do you see how each horizontal "layer" of the pyramid forms a klystron wave guide? However, since there are TWO pyramids oriented at 90 degrees to each other, the vertical pyramid is also a klystron. Look at the half-capstone triangle (BCD). Here C represents the velocity of light [(c) = 3x10^8 m/s] of an electromagnetic beam that enters the apex from directly above. As the beam pulses down the central column, its orthogonal wave front pulses down the apothem at the superluminal phase velocity of (Vp) = 1.27 (c) = 3.81x10^8 m/s. At the same time the wave front expands away from the central column at the group velocity of (Vg) = (c) / 1.27 = 2.36x10^8 m/s.

Now go back to the "horizontal" klystron. If we extend the "layer" horizontally, the light beam will zigzag along between the parallel layers. But what if the sides of the pyramid and the central column act as mirrors, as klystron walls? Let' s follow beam (C) in the third triangle from the apex in the drawing. Heading outward, it bounces back off the apothem wall, returning the way it came. When it reaches the central wall, it strikes at an incident angle of 50 degrees (40 degrees from normal) and then reflects off at the same angle of 50 degrees, which drives it directly to the base where (H) and (F) converge. If there is no mirror in the center, then the beam proceeds to the opposite corner of the base. A beam coming in horizontally along path (B) at the base of the "capstone" bounces off the opposite apothem wall at 50 degrees. It then proceeds to the center of the base and forms a nice isosceles triangle with the apothem wall (from base to point of reflection) and the base (from corner to center).



This fractal klystron can be used in various ways to adjust the scale of an EM signal. The scaling proceeds in the same manner as the increments of Planck' s constant, except that the increments are logarithmic instead of linear. The relationship between the phi scaling and what I call the "shofar" cornucopia scaling, which we introduce below, deserve careful study.