Chapter 3. Diagonalizing Over Infinity

Pure mathematics is an art form dedicated to constructing elegant structures in Mental Space. Because of its uncanny ability to reflect the structures in World Space, mathematics has also become the favorite tool of the physicist. In this chapter we will play with some mathematical ideas and develop them in creative ways. Then we'll begin to explore them as models of World Space experiences. We will come up with some tools for playing with physics that we will apply in later chapters in remarkable ways.

In Chapter 1 we briefly mentioned Georg Cantor's proof of the uncountability of the real set. This has a lot of bearing on the subject of continuity, which we have already touched on. The nature of the real set and continuity are essential to the way mathematics is done in a great deal of physics. The viability of the calculus and differential equations is all based on these notions. Therefore, we'll begin our discussion by taking a closer look at what Cantor was exploring.

Cantor began to study the nature of the infinite in mathematics. The physical world, while rich in multiplicity, does not seem to include infinitudes of anything physical. However, Mental Space appears to have no such problem, and mathematics is a tool for studying "precise" ideas about infinity. Even the simple natural numbers form an infinite set. It is open-ended and you can just keep on counting "forever".

Cantor wondered whether all infinite sets were the same size. Before that everyone pretty well assumed that infinity was just infinity, and it did not come in different sizes. So Cantor developed some techniques for studying infinity. His first tool was simply to map sets one-to-one. He used as his "metric", the set of natural numbers (N). He assumed that this would be the simplest form of infinity.

What happens if we just take half of the natural numbers, say the even numbers. Do we get half an infinity? Cantor lined up the two sets and found that they mapped one-to-one.

* 1, 2, 3, 4, 5, 6, ...
* 2, 4, 6, 8, 10, 12, ...

Interesting. Half an infinity equals a whole infinity. Infinite sets obviously do not follow the ordinary rules of arithmetic.

What about the integers (Z)? All the positive numbers plus all the negative numbers should equal two infinities, right? Wrong.

Cantor found that by folding the list in half he could map the positive and negative integers to the natural numbers.

* 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,......
* 1, -1, 2, -2, 3, -3, 4, -4, 5, -5,.....

So two times infinity also equals infinity.

Cantor went on to look at the rational numbers (Q). Here seemed to be a problem. The rationals must be more numerous than the natural numbers. They form a ratio of one infinity to another, so we have two infinities of natural numbers interacting: (m / n). Cantor organized the rationals into a neat array.

* 1/1 1/2 1/3 1/4 1/5 ......
* 2/1 2/2 2/3 2/4 2/5 ......
* 3/1 3/2 3/3 3/4 3/5 .....
* 4/1 4/2 4/3 4/4 4/5 ....
* 5/1 5/2 5/3 5/4 5/5 ....
* ....

If you count forever down the first row, you' ll never even finish the first row. How can they be countable? But Cantor noticed that by counting diagonally over his array he could map the rationals to the natural numbers.

* 1. (1/1), 2. (2/1), 3. (1/2), 4. (1/3), 5. (2/2),
6. (3/1), 7. (4/1), 8. (3/2), 9. (2/3), 10. (1/4), ...

Bingo!! He had counted the rational set (Q), and it was the same size as (N). He had also found a creative way of counting.

What about the real numbers (R)? Cantor could organize the set (Q) so that it could be written in an orderly fashion. But nobody has yet been able to write down the set (R) in a "well-ordered" fashion, even though Zermelo has shown that it is theoretically possible to do so. So the best Cantor could do was assume that there was some way to compose them into a list.

Once he had a "list" he found that, by adapting his "diagonalizing" technique in another creative way, he could generate a number that was not on the list!! This seemed to contradict the assumption that he had already compiled a complete list. His conclusion was that there is no way to get a complete list of (R), and therefore (R) is bigger than (N) and therefore also uncountable.

Let' s take a look at Cantor' s "proof". It' s actually a kind of "demonstration." This mechanical demonstration aspect makes it particularly interesting from the standpoint of physics. Here is a version of Cantor' s demonstration based on Eves and Newsom, p. 255-256. Our purpose in looking at this demonstration is not to critique the proof, but to play with Cantor' s ingenious technology.

Theorem: The set of all real numbers in the interval $(0<x<1)$ is non-denumerable.

1. Assume the set is denumerable.
2. List the numbers in the sequence $\{P_1, P_2, P_3, ...\}$.
3. Each (P_i) can be represented uniquely as an infinite decimal.
4. Form the sequence into an array:

$P_1 = 0.a_{11}\ a_{12}\ a_{13} \ldots$
$P_2 = 0.a_{21}\ a_{22}\ a_{23} \ldots$
$P_3 = 0.a_{31}\ a_{32}\ a_{33} \ldots$
....

(Each a_{ij} represents the digits 0 or 1 exclusively. We'll use the binary number system for this version of the proof.)

5. We can construct a number $P_x = 0.b_1\ b_2\ b_3 \ldots$ in which $b_k = 0$ if $a_{kk} = 1$, and $b_k = 1$ if $a_{kk} = 0$, for $k = 1, 2, 3, ..., n, ...$
6. Such a number clearly lies between 0 and 1 and differs from (P_1) in the first decimal place, (P_2) in the second place, (P_3) in the third place, and so on.
7. Thus the original assumption is untenable, and the set is nondenumerable.

QED!?

Cantor's list seems longer than his indices, but so does his list of rationals, and that is countable. Cantor's assumption is that he can somehow create a complete list, even though he can not figure out how to organize the set the way we do with natural numbers and integers. He does not provide an algorithm for generating his list the way he does for the integers and the rationals. Nor does he offer any proof that the list is complete. Thus his claim that his newly generated number is not on the list is questionable, because we can't really be sure his list is complete.

This is a real mind-boggler, because Cantor gives you a demo that **looks** very much like a complete list with nice indices that run in numerical order. Nevertheless it is probably a bogus list because only the indices are orderly, and the list's internal content is totally helter-skelter and lacking in algorithm. We have no way to know it is complete other than Cantor's assertion that it is so. This is how magicians mislead people when they do illusions. If his list is really NOT complete, then the number that he generates by diagonalizing may indeed be definitely NOT on his list, but can be just a number that he missed in the list, not a new decimal.

If you make the list helter-skelter without an algorithm you can never be sure it is complete! Cantor asserts that his list is complete, and assumes that it is, and uses indices to convince you into believing him!! I think this hidden assumption is such an important part of his demonstration that he must clearly prove that his list is complete for his nondenumerability proof to stand. Only then can we be sure that there is a contradiction.

One way to make sure the list is complete is to provide an algorithm for checking the list' s actual content, not its indices. Of course, an algorithm is equivalent to a counting method, so by providing an algorithm Cantor immediately would admit that irrationals are countable after all. All Cantor does is shuffle indices like a shell game. You don' t know what' s behind them.

So, can we organize our list systematically to make sure we' ve got them all? There are many ways to organize the list. Here is probably the most obvious one. It' s simply a mirror image of the sequential enumeration of the binary natural numbers flipped to the right of the decimal point and padded out to infinity with 0' s from the point beyond which there are no more 1' s. This mirror image list is not sequentially ordered by the dyadic relations, < or >, but has its own logical sequence.
0.0000...
0.10000....
0.010000...
0.110000...
0.0010000....
0.10100000...
0.011000000...
0.1110000000...
0.00010000000...
...
.....
0.111111111.....

This list includes every number from 0 to 1, right? (Recall that 0.1111... = 1.00000.... You may not agree. You may say that these numbers all have infinite 0 tails after them. There are decimals that have 1' s scattered through them all the way to infinity. This is a viewpoint. I think it is like saying that you don' t like to count the rationals diagonally. If you insist on counting Cantor' s array of rationals horizontally or vertically, then you CAN' T map the list to (N). If you allow diagonal counting, then (Q) maps to (N) with no problem. It is a matter of viewpoint. In our list the 1' s travel to the right slower than the 0' s, but they both eventually get to infinity. It' s like a tortoise and hare race where the 0' s and 1' s start out together, then the 0' s race out way ahead, but the 1' s keep plodding on and eventually "catch up". At infinity it turns out to be a tie. Another way of looking at it is to say that the non-local 0' s are already at infinity waiting forever, while the 1' s plod their way across the local numbers until they get to infinity too. At any rate, whether you believe the list is complete or not is not important to our discussion. We want to play with the process of diagonalizing that unfolds with THIS list.

We disallow the first number, 0. The final number (0.11111...) is the limit of our list at infinity. It is also an illegal number that is not on the list since it is equivalent to 1, a number which is not between 0 and 1. So the list goes from 0.100... to as close to .111111111... as you please, but never quite gets there. But that' s OK, since it' s an infinite list, just like the natural numbers. For purposes of organizational clarity I' ll leave the two book end limits 0 and 1 there for reference.

So if we exclude both 0.111... and 0.00000... from our list, leaving them there just as references, like we did when we wrote our definition of the list ($0<x<1$), where all the *x*'s form the list, we get:

(0.0000...)
0.10000....
0.010000...
0.110000...
0.0010000....
0.10100000...
0.011000000...
0.1110000000...
0.00010000000...
.....
.....
(0.111111111.....)

Our diagonalized number becomes 0.001111111111....
According to our rules disallowing infinite strings of 1's, this decimal is thus equivalent to 0.010000..., which is obviously the second number on our list!! Fortunately it's not way down in the list, though it could be, depending on how we organized things.

Our list satisfies Cantor's criteria for such an infinite list. Whether it is a complete list of all of (R) is another question. Every (Pi) in the set $\{0<Pi<1\}$ consists of an infinite string of (a_{ij}) digits to the right of the decimal that are either 0 or 1. We included every possible number of the pattern $\{0<Pi<1\}$ in our infinite list in an orderly fashion. And we diagonalized by his rules: We constructed a number

* (P_x) = $0.b_1\ b_2\ b_3$ . . . in which
* (b_k) = 0 if a_{kk} = 1, and
* b_k = 1 if a_{kk} = 0, for k = 1, 2, 3, ..., n, ...

Our number (Px) differs from (P_1) in the first decimal place, from (P_2) in the second place, from (P_3) in the third place, and so on ad infinitum. Yet when we diagonalized, we didn't get a new number, we got one that's clearly **on the list**. WHY????

There are two reasons.

First, there's the serious notational problem in the decimal system that we mentioned earlier. In binaries this shows up as the overlap between numbers with infinite 0 tails and numbers with infinite 1 tails. We followed the rule to convert the infinite 1 tailed numbers into infinite 0 tailed numbers, but that just gave us a duplicate number that was clearly already on the list.

Second, binary and base (n>1) numbers are more compact than unary (base n=1) "notch" numbers. In the way that we have organized the list, the 1's move out to the right as we go through the list more slowly than the "unary" diagonal digits move to the right as we go down the list. So, as we go to infinity, we diagonalize the whole infinite list in which each number is unique, but we always have a 0 at the diagonalizing digit. The "last" digit at "infinity" is the 1 that goes on the "end" of the infinite string of 1's to make the bookend limit of 1 that marks the termination of the list.

With natural numbers you can use mathematical induction to show that you have covered an entire infinite list. If (P_n) is some proposition defined for all natural numbers (n), and if (P_1) is true and [$P_{(k+1)}$] is also true, then (P_n) is true for the whole list of natural numbers, even though we have an infinite list. We can't do this with Cantor's list as it is constructed, or with our list.

However, since we have constructed our list of binary decimals via an algorithm that just mirror images the binary natural numbers (adding infinite 0 tails on to all of them), we can use "mirror reflection" mathematical induction to cover the whole list. The list of binary decimals is just a mirror map of the binary natural numbers. Even if you argue that our list is not complete for all infinite decimals, it is made totally of infinite decimals and is infinitely long. The fact that right off the bat we get duplicates that are already ON the list is an interesting situation given Cantor's claims for his proof.

We can construct an infinite number of these monkey wrench cases for Cantor. For example, take the list, which is identical in content to the above list, but slightly rearranged:

(0.0000...)
0.010000....
0.100000...
0.110000...
0.0010000....
0.10100000...
0.011000000...
0.1110000000...
0.00010000000...
.....
.....
(0.111111111.....)

The diagonal becomes 0.000000000... and its flip is 0.111111.... Both of these numbers are well known. They are 0 and 1. They are outside the range of our defined set. Here's another case:

(0.0000...)
0.10000....
0.1100000...
0.010000...

0.0010000....
0.10100000...
0.011000000...
0.1110000000...
0.00010000000...
.....
.....
(0.111111111.....)

This one gives the same result as my first example. (0.0100...) You can generate as many examples as you like, and, as long as you keep the diagonal digit ahead of the creeping 1's after some point on the list, I think you'll find the flipped number will be on the list or will have the value 0 or 1.

Now let's start to think about this little mathematical game in terms of physics.

Cantor's dummy indexed decimals don't really let you check his list, because you do not know the values of his (aij)s. They are just probabilities. Each digit can be a 0 or a 1 with 50% (or .5) probability. This is just like the quantum problem of the observer not knowing whether the electron spin is up or down, a photon is polarized this way or that, or the Schroedinger cat is dead or alive.

The quantum wave function is a dummy indexed "number". A particle is an actual number. When you produce an actual list, such as we did in our little experiment above, and look at the actual numbers, then you know which way each digit is, 0 or 1 in each position. In the case above, where we can check the list, we find that the diagonalized number is indeed on the list or is a limit, **substantiating** our claim that our list is indeed complete whether or not it has anything to do with (R). And it is not possible to construct a new number that is not on the list. When we "collapse the wave function" by actually observing the flipped number from a real list, it falls into place somewhere on the list just like an observed particle appears somewhere in the range of its wave function. Remember that Cantor never actually produced a single number on his list or a single actual diagonalized decimal or flipped diagonal. Before we look at a flipped diagonal, this dummy diagonal made of a string of indices hovers somewhere in a transcendental land of all possibilities outside the dummy list that we haven't really looked at either. This is what I mean by Observer Physics (and Observer Math.) Whether you look or don't look at something makes a world of difference.

If Cantor's diagonalization does produce one (and only one) extra number not on the list, it forms the 'limit' of the range of the list. He did not put his list in ascending or descending sequence by value. Therefore the sequence of items on his list of reals, if it is indeed complete, and REGARDLESS OF THE INDIVIDUAL NUMERICAL VALUES of the numbers on the list (which do not matter here), provides a ONE-TO-ONE MAPPING to the points in a line. The extra number that seems to pop out from diagonal flipping is just the end point of the line. It is the limit. Forget its numerical value. (Forget the location of the particle in space/time.) It's like flipping a coin.

Careful study of this phenomenon reveals the secrets underlying the theory of limits and the calculus as well as how quantum mechanics works.

So if we take Cantor's viewpoint and assume he is right, we get a line plus one point. Rediagonalizing the same list with binaries gives you your original diagonal, not any more new numbers. That's why I use binaries. It makes this point clear. The situation with other bases is slightly more complicated, but essentially the same. Of course, we can shuffle the list and do another diagonal and get another flipped number. But this is like drawing a different line with a different end point. Any two lines, regardless of size or location, are equivalent topologically as sets of points. The numerical values are irrelevant in Cantor's dummy system that is not ordered. It is just a set of points.

Cantor's list of real decimals is like a sequence of dots, each dot with a companion gap, the space between any two numbers on the list. By virtue of being a list, by definition, the set is denumerable. It is nonsense to talk of a nondenumerable list and uncountable numbers. To me an "uncountable" number suggests a variable, like (x). The space of a gap is a variable. But not the gap itself. It belongs to the category of noncount nouns, like air and water. The number of gaps in the list equals the number of dots unless we add one more dot to represent the limit -- the end of the line. That's what the flipped diagonal represents. When you change the base, you are simply shifting the line into a plane with one dimension "continuous" and the second "digital". If you use the list as your base, then you have a true Euclidean plane, with both dimensions "continuous".

This discussion also gets into the interesting territory of Zermelo's Well-ordering Theorem and its equivalent, the Axiom of Choice. No one has been able to well-order the above list of decimals or similar sets -- i.e.,produce an algorithm to put them in (a < b) order. You'll note that my sample list above is NOT well ordered. There's a nice exercise for someone: Using binaries, well order the infinite decimals between 0 and 1. Zermelo proved it can be done!! (Hint: To do it we first have to resolve the notational problem of repeating 1's that interferes with constructing a good algorithm.)

When I comment on Cantor's work, I in no way impune his genius as a mathematician. He is one of the greats. His diagonal system is a profound invention, a truly simple and powerful technology. It is a very useful tool, though not necessarily in the way he intended it. This unexpected twist of evolution often happens with discoveries and inventions, and other mathematicians have used the diagonal technique in various ways. It embodies the fundamental principle elucidated by Maharishi in his Science of Creative Intelligence that any truly powerful technology must be capable of transcending itself. The diagonal technique itself is an excellent model that can be applied in various ways. It is simple, elegant, natural, and robust -- all nice qualities for a scientific model.

As an analogy we can compare Cantor's diagonalizing to the process of TM. Consider each number on the list to be a thought in the mind of a meditator. Each succeeding thought is like the next number on the list. As the meditator moves down the list doing his meditation, he is moving in the direction of the infinite. At each succeeding number his attention shifts to subtler and subtler (akk)s -- aspects of thought -- represented in the number by the smaller and smaller, finer and finer digits of the decimal. When the meditator gets to the "end" of the list, his attention transcends the list and finds itself OUTSIDE the list. He is not on any number. He is at infinity. Suddenly a new number appears. This number is not on the list. It is a new creative thought that arises from the source of thought, or source of numbers, beyond the list.

Truly an ingenious technology, don' t you think? And it is intimately related to the calculus and other mathematical tools that are essential to modern physics..

As my lists showed, Cantor' s system really does produce numbers that are NOT on the list. All the examples I gave did so. And we did not guarantee that any one of our lists was complete set of (R), only that each was infinite. However, in each case the "new" number would always transform itself magically into a number that was already in the list or to the limit of the list. This is like the mind of the meditator automatically integrating itself back into ordinary life again after his transcendental experience during meditation.

Now let' s explore the "analogy" I mentioned between numbers and quantum mechanics a bit more. Math provides wonderful models for looking at the world. As we commonly see with computers, numbers can be interpreted in many ways. One of the interpretations is graphical. Numbers can form bitmap graphics.

Infinite decimals are like infinite digital wave forms.

As we shift to the right from the decimal point in some number on our Cantor list, we can imagine we are moving in space/time farther and farther away from a starting point. Things seem to get smaller as they get "farther away". Or we can imagine that we are zooming in to finer and finer levels of magnification of an object. Objects seem to get larger and larger as we zoom in. So if our attention is on objects, "zooming out" makes objects look smaller and "zooming in" makes objects look bigger. However, if our attention is on space, then zooming out makes the space we are aware of appear bigger, and zooming in makes the space we are aware of appear smaller. This is true of course only if we have some objects as reference that we assume are holding "still". Cantor' s list is a reference frame, and his indices allow us to locate ourselves anywhere in the space enclosed by this reference frame.

As you might guess, the subject of reference frames is quite important in Observer Physics. OP is a more general viewpoint than conventional physics, but conventional physics operates nicely as a subset of OP.

The same sort of thing happens with time frames. Size is an illusion that is relative to the Observer and the reference frames he selects. Measurement is a mapping of two arbitrary systems that may be in the same frame or different frames of observation. The Observer does all the "zooming" with his attention. We do zooming in, zooming out, shifting, panning, focusing, unfocusing, dividing, integrating, fixating, unfixating, plus a few other tricks with our attention. Attention management is what physics and living is all about. Underlying physics is the scientific study of consciousness, awareness, viewpoint, attention, belief structures, and the role of the Observer as witness and/or participant. Palmer's **ReSurfacing** is a handbook of attention management, well worth exploring.

Exercise: Do #18 "Viewpoints" and then #19 "This and That" in **ReSurfacing.**

In our decimal analogy empty space can be represented by 0.00000..... From quantum mechanics and modern physics we know that in our real world there is no such thing as 0.0000.... because there is always latent mass-energy in the ground state. There is also no such thing as 0.111111....., even as a probability. So anything and everything at any scale in the universe can be represented by some decimal between 0 and 1 with proper topological mapping.

There are as many real decimals as points in a continuous line interval. Mathematicians have shown that, oddly enough, there are as many of points between any two intervals of any shaped line, straight or curved, including also planes and n-dimensional spaces of any degree. This means that mental space, and probably also physical space, is a fractal hologram that maps one-to-one at any scale or topological distortion, and it agrees with the above analysis of relativity of zooming, something that Einstein has explored nicely. Where it breaks down is when different laws kick in at various different scales, showing that physical space is not a purely fractal space, but is quantized.

Periodic decimals, such as 0.01010101...., or 0.0111011101110111..., etc. are like various periodic EM wave forms or other periodic waves. The former is a pure infinite sine wave, assuming that we do not scale. Otherwise it is a damped wave. If we do not scale the decimals, a damped wave looks like this: 0.11111001111001110011001001000000...)
A number like 0.0111010111010111011101... is an interference of two or more sine waves. Periodic decimals of the form 0.1000... or 0.0011010000...., or the damped wave, and other cases represent composite sine waves that appear to be localized "bumps" or particles and various other finite wave shapes. This analogy gives us a kind of digital Fourier analysis. Since computers can interpret strings of 1's and 0's as graphics, this is a very nice analogy.

Also, we know from quantum mechanics that Planck's constant sets a limit to the resolution of the physical world, at least from our current viewpoint. That means our physical world is basically digital. But it is a curved, distorted digital world, because scaling goes on as you approach the limits. Therefore, it may be better to use the scaled decimals in some cases.

Now let's go back to our list of decimals using the "mirror image" of the natural binary numbers.

(0.000...)
0.1000...
0.01000...

0.11000...
0.00100...
0.101000...
0.111000...
0.00010000...
0.10010000...
.....
.....
(0.1111....)

If we let our attention scan down the list at some arbitrary rate, what we get is a "movie" (sequence of still frames) of a wave that appears to propagate from the left to the right into the empty space of "0000....". A tiny impulse appears and then fills into a solid wave that pops and pushes another tiny impulse forward that then fills and pops, and so on to "infinity" (i.e. the limit.)

In our analogy this propagating wave movie might represent the time evolution of a qwiff. It is not a numerical sequence, it is an algorithmic graphic sequential evolution. The particular base that we choose determines the relative "speed" in digits/decimal-place with which the wave propagates into "space". At any arbitrary base this speed has a fixed speed limit that can not be exceeded. If you are required to fill each possibility before moving on, then you can not go faster or slower. You must go at that speed. This is like the speed of light in a particular medium. The medium is the base. Binary is like (c) in "empty" ground state quantum space. Other media appear in other bases. Unary would be a theoretical top speed, but we can' t make decimals with unary, because, by definition, a unary base only deals with units or wholenesses, not parts of things.

EM waves always propagate at (c), even in various media. The waves just get gnarly in denser media. We live in at least a dualistic universe. If the base (n > 2), then we are just complicating the space with gnarly twists to slow down the apparent propagation of our wave, which still really goes at (c).

When we run Cantor' s diagonal technique on our list, we get a "number" that is NOT on the infinite list. For example, let' s say the diagonal from our sample list is 0.11000.... (which is obviously on the list.) But when we flip this number according to Cantor' s rule, we get 0.0011111.... As we pointed out, this "illegal" number collapses (qwiff pops) into 0.01000..., which IS a number in a specific place on the list.

Now what is this weird Cantor "number" 0.0011111.... in our quantum space "model"? It represents a superluminal wave. It is a conjugate wave to 0.11000...!!! When this "wave" appears at the end of the list, it runs backwards instantly from infinity to drop into some spot in the list as soon as the observer identifies it!!!

In our analogy any number that degenerates into an infinite string of 1' s is "superluminal". It has jumped ahead of the observer' s propagating wave front to infinity (the limit). We know that it has, because it comes from beyond the list. The wave moves all the way to infinity (the end of our infinite list), and then pops and reflects back from infinity instantaneously (FTL -- Faster Than Light!!) to some localized position in the list. The two numbers 0.11000... and 0.0011111... are conjugate numbers!! They are just like two photons reflected in a laser cavity that has generated phase conjugation. They are the same wave, resonating in retarded and advanced modes in a bubble of quantum space/time.

Cantor was way ahead of his time, and didn' t realize it! He created a nice model for qwiff pops and phase conjugation before anyone ever dreamed of such ideas.

Before taking leave of Cantor, let' s take one more look at the completeness of our list. Irregardless of the countability issue, this consideration will allow us to bring up another important principle of Observer Physics. Someone may object that our list is not complete because there remains a huge class of periodic and non-periodic decimals that have 1' s scattered throughout, all the way to "infinity". For example,

* 0.101010101010101.......

Our list always ended in 0' s. We took the viewpoint that the 1' s also make it all the way to infinity. But let' s accept this criticism that there is a huge list of numbers with 1' s that inherently can be found scattered "all the way" throughout the decimal to infinity and play with it a little.

If Cantor can give us a complete list without telling us at all how he did it, then perhaps we can give ourselves a magic sieve that will separate the infinite decimals into two sub-lists: those with 1' s scattered throughout them, and those that are finite decimals and stop having 1' s at some point and degenerate into infinite 0-tails. We' ll call the latter sub-list "degenerate" decimals, and the former sub-list non-degenerate decimals. Non-degenerates include both the infinite periodic and infinite non-periodic sequences so long as they have 1' s that scatter throughout the infinite sequence.

Let' s assume we' ve got all possible binary decimals -- degenerate and non-degenerate. We take our sub-list of degenerate decimals (any one of the sample versions I gave previously) and append it after the sub-list of non-degenerates. We get something like this:

0.1010101....
0.0001001001000...
0.111011101110110110...
....
...
0.1000000...
0.0100000...
0.1100000...
0.0010000...

.....
.....

It is clear that in the final sub-list the 1's will never catch up to the unary propagating diagonal because the second sub-list is degenerate. That means that however the diagonal is ordered as it passes through the first sub-list, the diagonal will always end in an infinite zero tail -- it will always be degenerate. Therefore its flipped Cantor number always will be of the form 0.(......)111111..., which means some long list of 1's and 0's (.....) followed by degeneration into an infinite tail of 1's.

Thus the Cantor number will always be a "conjugate" number that collapses into a degenerate decimal that IS found somewhere in the second (degenerate) sub-list. This suggests that the non-degenerate sub-list is really just a dummy list.

Our above set of two sub-lists covers the case of having all the decimals for sure. Or does it?

One might object that by organizing the decimals into two endless sub-lists one after the other that we have created a situation where we would never get to the second sub-list in our diagonalizing process. We have defined that our decimals are infinitely long. So we would expect both sublists to be infinitely long, since the unique properties of each list must extend to infinity. We are not separating the periodic and non-periodic infinite strings of scattered 1's. It's not necessary for our argument.

This situation is very much like the set of rational fractions. Arranging them in order by size or in other certain ways they are uncountable, because you keep going down infinite "dead-end alleys". But, as Cantor showed, if you make an array by ascending values of numerator and denominator arranged in rows and columns respectively and then zigzag diagonally starting at the upper left corner, you can nicely count this complete list.

1/1 2/1 3/1 4/1 5/1 . . .
1/2 2/2 3/2 4/2 5/2 . . .
1/3 2/3 3/3 4/3 5/3 . . .
. . . .

We count them: 1/1, 2/1, 1/2, 1/3, 2/2, 3/1, 4/1, 3/2, 2/3, . . . and get them all even though counting down any row or column you'd get stuck going on for ever.

We can do something like Cantor does with the fractions with our sub-lists of the real decimals. We can splice the two sub-lists together, alternating one and then the other on down a single list. For the moment we'll assume the two subsets are of EQUAL size. We'll come back to this important issue of sub-list length and deal with it in a moment. Then, when we diagonalize the spliced master list, we skip every other decimal, diagonalizing ONLY the degenerate sub-list. With this method we treat the non-degenerate decimals as "gap" numbers already "pre-labeled" by degenerates. In other words, the non-degenerates are mapped one-to-one to the degenerates. Then we diagonalize.

Why do we do this? Because we know that in this way for sure we will always get a number that is not on the total master list. We know that neither sub-list, by definition, has a tail that ends in an endless string of 1's only, because endless strings of 1's are "illegal". Non-degenerates always have 0's sprinkled throughout them at various periodic or non-periodic intervals. Diagonalizing the whole list, we can't guarantee a 1-tailed number will result. But by diagonalizing only the degenerate members, we guarantee a 1-tailed number that is not on the list, a "conjugate" Cantor degenerate number.

This shows us also clearly that the non-degenerate sub-list is really a dummy. To be sure our number is not on the combined list because it is not a degenerate number. We actually have to DISREGARD the non-degenerate portion. If we diagonalize the whole spliced list or the list with the degenerate sub-list coming in front of the non-degenerate sub-list, we get a number that has 1's scattered throughout. It seems like it is a "new" number not on the list, but we can never really be sure in this case since we have no way to check the non-degenerate list like we do in the methods I've mentioned above. We do know, however, that the "new" number in this case is NOT a degenerate. It must therefore be a non-degenerate since these two types cover all possible cases.

The conjugate number from diagonalizing the degenerate sub-list, though obviously NOT on either sub-list, actually always turns into a number on the degenerate list (or the limit of the whole list.) I suspect that the same is true of a diagonal of the non-degenerate sub-list only or of the whole list with non-degenerates coming after degenerates or of the case in which non-degenerates are more numerous than degenerates. In any of these cases you get a "conjugate" number that has 1's scattered throughout. It would have a conjugate partner, which might even be itself (with phase shift?) if it is periodic and symmetrical. (It might be a flipping of each digit, 1's to 0's and vice versa. After all, that's how Cantor does his diagonalizing trick -- he flips his diagonal, making a conjugate pair: e.g., 11000101100.... <--> 00111010011....) Every non-degenerate decimal has a conjugate pair (0's and 1's flipped) somewhere on the list -- its soul mate or twin flame.

In any case we know that our non-degenerate sub-list by our initial definition contains every infinite sequence of 1's and 0's that has 1's scattered among 0's throughout its whole infinite length. We assume that the diagonal itself before being flipped is definitely on the list. That is certainly the case for the degenerate sub-list. The "new" number must be on the non-degenerate list somewhere, since every non-degenerate has its soul mate somewhere in the list. You can not claim untenability of denumerability due to the number seeming to be "new", because you have no way to check the actual numbers on the list. And we have demonstrated with the infinite degenerate "sub-list" that, just because you generate a "new" number by flipping digits at every index, does not mean that the number is not ultimately found on the list. Thus we can not assume that the new number is really "new" for the non-degenerate sub-list when it is clearly not so for the degenerate sub-list.

We know that every flipped diagonal taken from a non-degenerate sub-list will have 1's scattered throughout. We also know that the non-degenerate sub-list contains all possible ways of scattering 1's throughout an infinite sequence. Thus our flipped diagonal must be on the list somewhere because it fits our only known defined data about the non-degenerate sub-list -- that the number is an infinite decimal that is non-degenerate and contains 1's scattered throughout. That's all we know. We can't say anything more about it in this case, because we lack a way of checking physically. Our viewpoint is localized, and the list is non-localized. If we define one additional quality of the non-degenerate sub-list: that it contains a complete set of conjugate pairs or soul mates, then we know the number will be on the list as the diagonal's soul mate -- by definition.

When we splice the two sub-lists and diagonalize, there are three possibilities. The two sub-lists can be equal in length, or there can be infinitely more degenerates, or there can be infinitely more non-degenerates. I don't see how there could be a finite difference in set size, though I might be wrong there. If the degenerates are equal or more numerous, then the diagonal has a tail of 1's and the Cantor number is a conjugate that is really just a degenerate already on the degenerate sub-list as we have seen. If the non-degenerates are more numerous, then the diagonal is a non-degenerate decimal, and the non-degenerates may or may not be countable depending on whether the "new" number is or is not actually found in the list. We have no way of knowing for sure as long as the Observer may not actually OBSERVE the numbers on the non-degenerate sub-list. Dummy indices are not actual numbers, they just represent the presence of a privileged non-local system to which we are denied access. The conjugate soul mate definition allows us a tiny "peek" at the list.

This Cantorian model is just like the role of the Observer in a quantum system. The secure single photon key transmission system being developed at Los Alamos is a good example of a practical application based on the statistics of an Observer collapsing photon wave functions. If an Observer intercepts and peeks at the transmission, he irrevocably changes the quantum statistics by his very act of observation. The unauthorized observer can not reconstruct the original or a fake signal without the receiver knowing that someone unauthorized has observed the signal and/or otherwise tampered with it. In a similar way the non-degenerate sub-list is secure from the prying eyes of localized individuals. Only non-localized individuals such as angels and ascended masters can verify data in that sub-list. Anything localized people look at turns into a degenerate pumpkin number: Bippity Boppity B000000....

Thus we can use OBSERVER PHYSICS and OBSERVER MATH to clarify subtle and profound issues in the foundations of science. The OBSERVER is critical to such processes, even in pure math. Maybe love and other methods of consciousness expansion provide ways by which people find soul mates that are non-locally separated.