The Great Einstein/de Broglie Velocity Equation

derivation and notes by
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$$(V_m)(V_i) = (c)(c).$$

This is the Great Velocity Equation. It is based on the revolutionary discoveries made by Albert Einstein and Louis de Broglie during the early years of the 20th century. This Velocity Equation is really a rewritten form of Einstein's famous energy/mass equation.

- $E = m\,c^2$

Actually Einstein's equation is meaningless because energy and mass are not observable without disturbing the system under observation. Any disturbance to the system changes the system and therefore nullifies the value of the observation. The extreme nature of this problem becomes clear in the field of particle physics. Therefore I have rewritten the Einstein formula using a few simple substitutions so that its component factors are all observable velocities. The "direction", frequency, and/or wavelength of a photon may change when it is disturbed by observation or other intervention, but the photon velocity remains constant.

The two Velocities on the left side of the equation represent variable velocities. The Velocity with subscript (m) represents Matter, anything with what physicists call "mass". The technical name for it is the "Group Velocity" (V_g). The Velocity with subscript (i) represents Imagination. This refers to images that we can form in the mind either by connecting observable images in some way or by connecting mental images in some way. The technical name for it is the "Phase Velocity" (V_p). Phase means that we observe an object with respect to some other reference object. This is the "connecting" that I mentioned. The velocity on the right side of the equation is light speed (c). It occurs twice and is often written as c-squared (c^2). Each c corresponds to one of the two variable velocities. One is the photon that moves from a physical object to the observer's eye. The other is the anti-photon that moves from the observer's eye to the physical object. In ordinary space the two seem to overlap, but they are really in different spaces. The multiplication of the two factors expresses their interaction. The photon moves forward in time from the object in the past to the observer in the present. The anti-photon moves backward in time from the observer in the present to the object in the past. The two appear to travel together most of the time, but under certain conditions they can split apart, distorting space/time. Under these conditions the values of the velocities appear to become different from the speed of light. This gives us the variable velocities on the left side of the equation.

The word "group" used by physicists is really misleading, because it implies that the phase velocity is not a group phenomenon. This is incorrect. The very definition of phase tells use that it is a group phenomenon. The reference object used to determine the phase is a second wave, even if it is only a point. Fourier analysis shows that such a point is a very complex wave phenomenon. The photon interaction is also a group interaction since it involves an emitter and an absorber [such as an electron] as the two terminals for its trajectory.

Einstein declared that objects in the "mass" condition could not equal or exceed light speed (c). Since the value of c never changes, the Velocity Equation describes a reciprocal relationship between the two variable Velocities. Thus, if we set c equal to unity as many physicists find convenient, and if Vm is some fraction of unity, then Vi must be the reciprocal of that fraction. For example, if Vm is 1/2 light speed, then Vi must be 2/1, or twice the speed of light: $(1/2)(2/1) = (1)(1) = 1$.

Einstein declared that we can only transmit information via the slower-than-light group velocity, the upper limit being the speed of light used for portions of the transmission. The equation shows that this is not true. Whatever information can be encoded in the "mass" variable will show up in a reciprocal form in the "image" variable. As I will demonstrate, both modes of communication are common in our lives. They simply work by different rules for the observer because of their reciprocal relationship.

Another astonishing result of this equation is that it demonstrates the value of meditation. If you bring the velocity of your body as close to a full stop as possible, then your imagination expands in a reciprocal fashion. The mind moves faster than light and can easily encompass the entire universe. You can easily do this. Simply sit quietly and then imagine that you are bigger than the universe. Bingo. There you are, outside the universe. Is it just your imagination? Of course it is. But why do you believe that your imagination is less real than your physical body? You can use your imagination to create things that are real. Try it. Imagine something creative and then make it real. Just a simple thing, like making an unusual motion or a funny doodle will do.

All phenomena in the universe can be described as oscillating waves. An oscillation has an amplitude, wavelength (λ), a frequency (v) or period ($T = 1 / v$), and a phase. Fourier showed that in general we can represent any function as the superposition of a set of periodic oscillations. If the oscillation has a time evolution, then it produces a train of waves that move along at a certain velocity, which is the wavelength times the frequency. Since this represents a displacement of the wave's phase through space over time, it is called the phase velocity.

* Phase Velocity: $(Vp) = (\lambda)(v) = (\lambda) / (T) = (\omega / k)$, where $(k) = 2\pi / \lambda$; $(\omega) = 2\pi / T$.

$(\pi = 3.1416.....)$

Of course, a group of interacting waves can also produce the appearance of phase waves. For example, the interaction of the two blades of a pair of scissors produces a phase wave for an observer when the scissors blades open or close on their pivot. This wave exists as a reality only in the imagination of the observer who views the two blades from a viewpoint in which they appear to interact. Actually the blades just pass by each other and need not even touch. Phase waves are a product of the imagination and have no speed limit other than what is defined by the system an observer decides to watch. However, Einstein claimed that matter waves can only move at speeds slower than the speed of light. As we show in the notes below, this limitation only holds by accepting the limiting definitions of what an observer observes as and is not an inherent speed limit. Einstein said that the rule holds for objects with mass, but there is no way to determine the "mass" of an object without interacting with it. Thus it is hard to tell exactly what an object with mass is and what an object of imagination is. Objects of imagination are simply images that an observer defines in his mind and do not inherently have any mass. When we watch cars moving on TV, we form images in our minds based on light flashes on a screen. We believe they are just images, somehow different from the images we see of cars moving on the street. But there is no way to really tell the difference without going out and touching a car. We simply take it on faith. We will soon have virtual reality technologies that reproduce the sense of touch so you can reach out and touch the car on the TV screen. So where is the crossover between Matter and Imagination?

In 1924 de Broglie showed that any particle of matter such as an electron also could be interpreted as a wave packet -- that is, a superposition of pure periodic oscillations that generates the appearance of a lump that behaves like a localized particle. His reasoning was based on the Einstein-Planck relations that show how the energy of a photon depends on its frequency times the universal constant of wave resolution (h) and Einstein's famous equation showing the mass-energy relation [$E = M c$^2].

* $E = (h)(v)$.
* $E = M c$^2.
* $M c = (h)(v) / c$.
* $v / c = 1 / \lambda$
* $M c = h / \lambda$
* $\lambda = h / M c$.

Taking (Me) as the mass of the electron and letting (c) become the velocity of the electron (Ve), we get the de Broglie wavelength of the electron (λe).

* $\lambda e = h / Me\ Ve$.

This can be expressed as the group velocity of the wave packet forming the electron (Vg).

* $Vg = h / Me\ \lambda e$.

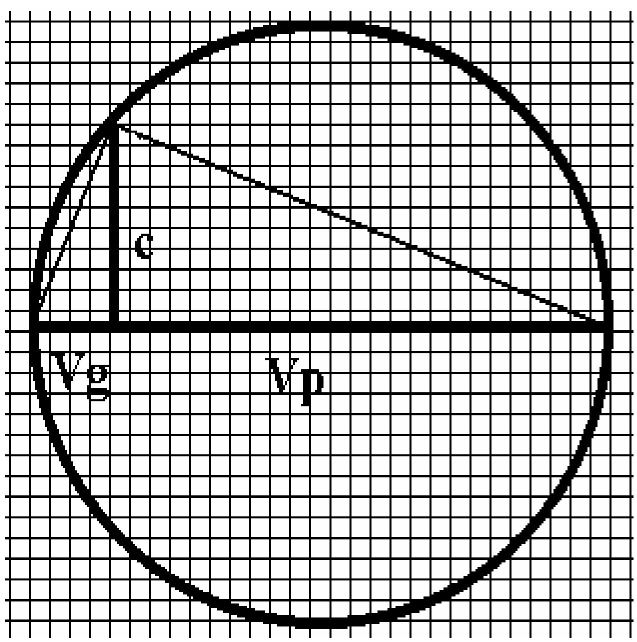
Substituting the mass of the electron in terms of its energy, we get the wave packet's Group Velocity (Vg), which simplifies to the Velocity Equation.

* $Vg = (h / \lambda)(c$^2 $/ h v) = c$^2 $/ \lambda v = c$^2 $/ Vp$.
* $(Vg)(Vp) = c$^2.

In the case of light propagating in a vacuum, both (Vg) and (Vp) have the value of (c). But in the case of light passing through a dispersive medium, or a klystron, or the electron, or any other particle of matter, the group velocity of the wave packet -- the velocity of the packet's maximum amplitude, an illusion produced by the interference of the various superposed phase waves (*Dω* / *Dk*) -- is less than the speed of light. Therefore, there must be phase waves associated with the particle's wave packet, and these must move faster than light. Most physicists prefer to disregard the curious superluminal phase waves as irrelevant. However, I believe that this equation actually constitutes a precise definition of cosmic consciousness (*cc*).

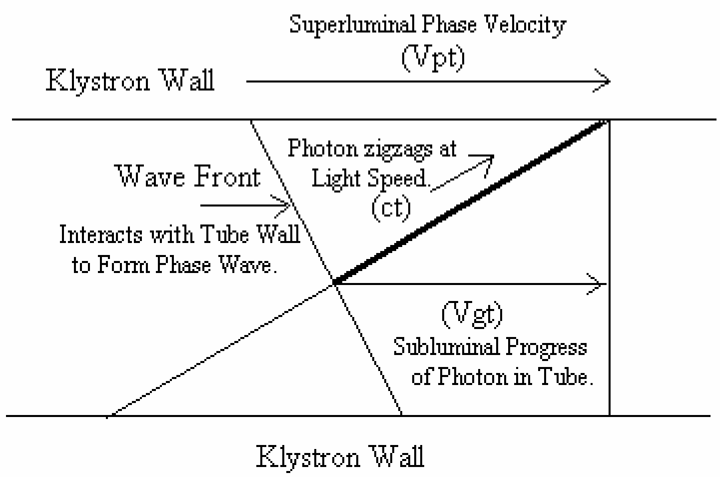
The group wave represents the object of perception. The phase wave is the perceiver's consciousness, his imagination. Multiplication of the two represents their interaction, the process of perception. One (*c*) is the retarded wave, and the other (*c*) is the advanced wave. The equation tells us that, when we detect the material particle (the so-called "observable" group wave packet), we tend to miss the superluminal phase waves associated with it. In cosmic consciousness we learn to appreciate the dynamics of the phase wave as well as the group wave.

In sections 4 and 5 of his lecture number 48, entitled "Beats", Richard Feynman discusses localized wave trains and probability amplitudes for particles. Here he considers the relationship among (*c*), (Vg), and (*Vp*), analyzing it in a different way, but comes up with the same velocity relationship -- although he does not write out the equation in the same way we do.



Here is another way of representing The Great Velocity Equation. Draw a circle and then draw a diameter through the center. Select any point along the diameter [except the end points] and erect a perpendicular. There are actually two perpendiculars, one on each side of the diameter. Label the perpendicular *c*. That represents the speed of light. Then label the shorter segment of the diameter *Vg*. This stands for the Group Velocity that is associated with objects that have "mass". Label the longer segment of the diameter *Vp*. This stands for the Phase Velocity that is associated with the imagination of the observer. Phase is an arbitrary assignment of a relationship that is made by the observer. It therefore has no specific speed limit. However, in this chart there is a phase velocity that is determined by the mass we choose to observe relative to the speed of light. From this diagram it is clear that *Vg* will always be smaller than *Vp* [by our definition], and will always be smaller than c unless the diameter is divided exactly in the middle. In that case *Vg* equals *Vp*, and the two velocities also equal *c*. *Vp* is always larger than or equal to *c*. It is also clear from this diagram that the speed of light is not really constant, since it depends on the relationship between *Vg* and *Vp*. As *Vg* shrinks, *c* also shrinks, while *Vp* expands. If *Vg* exactly equals 0, which is not possible in the physical world, *c* appears infinite and there is no way to evaluate *Vp*, which becomes equal to the diameter. But if *Vg* is very small, the *Vp* appears extremely large compared to *c*. In any case the relationship always remains constant due to the similarity of the two right triangles formed by the perpendicular: $(Vg / c) = (c / Vp)$. Thus the Velocity Equation holds: $(Vg)(Vp) = c^2$.

Here is another way to visualize the Velocity Equation. In this case we examine a klystron, which is a rectangular tube used to guide microwaves.



The klystron wave guide used in microwave technology clearly shows the relationships of the three wave types. (*Vg*) is the group velocity, (*Vp*) is the phase velocity, and (*c*) is the photon moving at the speed of light. We convert velocities into relative distances by multiplying each velocity by the same unit of time (*t*) which then cancels out of the equation.

In the klystron tube, (*Vg*) and (*Vp*) are parallel motions along the direction of the tube, but (*c*) zigzags reflecting back and forth from wall to wall as the photon proceeds down the tube. (*Vg*) represents the photon's net forward progress. (*Vp*) represents the interaction of the photon's wave front with the tube wall. This interaction is a non-local phenomenon. The wave front is always normal to the photon trajectory. You can see from the geometry that (*Vp*) is always greater than (*c*), and (*c*) is always greater than (*Vg*) -- except at the moment when the photon bounces off the tube wall. In the infinitesimal instant of the photon's interaction with the wall, (*c*) appears to drop to 0 because the photon is momentarily absorbed and then re-emitted by an electron in the tube wall. At the moment of interaction between photon and electron the photon's net momentum shifts direction and reflects back out from the wall. An important principle for electromagnetic wave guides is that all of these velocities are interactive and can not stand alone. The phase velocity depends on the interaction of the wave front and the tube wall, the group velocity depends on the interaction of the photon with the tube wall, and the speed of light depends on the interaction between two terminal points such as electrons, an emitter and an absorber. Without terminals a photon can not move or even manifest. The curious thing about the relationship between the photon and its terminals is that it always moves at (c) relative to its terminals, regardless of their positions or any other relative motions. It balances the differences by shifting its apparent wavelength rather than its speed. Since the photon actually radiates in all directions from its source, what we observe as the wave moving through the klystron is actually the resultant of the interference of all the radiation probabilities of the photon in the tube. It becomes observable as the path of least action for the particular frequency in the tube.

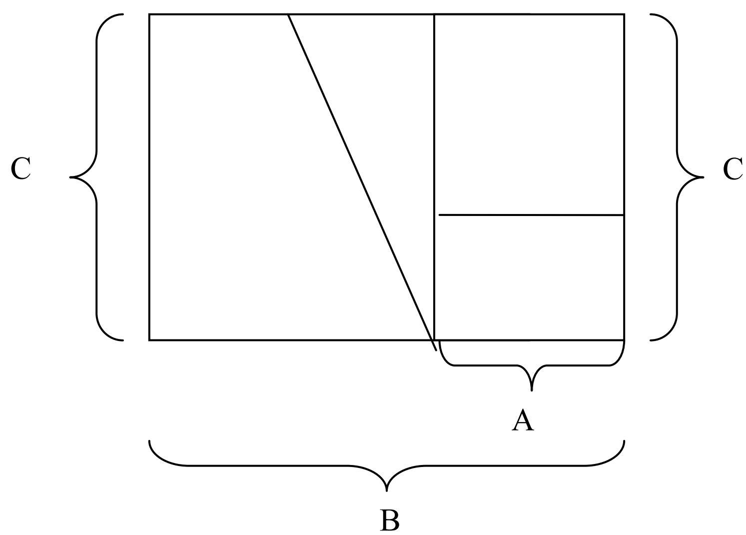
The basic relationship between group and phase velocities is orthogonal because a wave's "front" is orthogonal to its direction. In the klystron the two similar triangles are oriented in a flipped fashion and we use one leg and the hypotenuse for our equal ratios instead of the two orthogonal legs of the two similar right triangles. [You can draw the diagram, cut out the triangles, and reorient them to see this.] If the wave front in the klystron is parallel to the tube, the photon wave just bounces back and forth between the walls and will not travel down the tube. Its group velocity is 0. The whole tube becomes the phase wave at infinite velocity. However the phase wave initially has to erect itself at light speed down the tube. Then it becomes a standing wave. In the circle diagram example the large Vp triangle is tilted 90 degrees with respect to its smaller similar Vg cousin. The velocity relationship is always the ratio of corresponding sides of similar right triangles. The group velocity is temporal (less than light speed), and the phase velocity is spatial (greater than light speed). The group velocity viewpoint is like standing on the ground and watching cars arrive at a destination one at a time after they have traveled along a highway [serial transmission]. The phase velocity viewpoint is like going up high in a helicopter so you can see all the cars spaced out along the road in a single glance [parallel transmission]. The entire light field that holds the information about the cars that are on the road arrives at your eye in one single snapshot view. The viewpoint shift between these two observations is basically 90 degrees. But it also includes a shift in perspective. In the case of serial transmission, the observer sits at one end of a transmission line and is small and separated from the transmission source. In the case of parallel transmission the observer is above the transmission line and has an expanded view that is larger than the transmission line and embraces the whole system including source, transmission, and reception of the data. Therefore, in order to implement faster-than-light transmission on a cosmic scale, the observer must have a perspective that is transcendental to the cosmos. Thus all information becomes internal information. It goes "faster than light" because it essentially does not "go" anywhere.

Exercise: Compare talking on the telephone to looking at a photograph. The telephone transmits in serial fashion, and the photograph transmits in parallel fashion. You hear one word at a time to get the telephone message. You see the whole data transmission of the photo in a single glance.

Exercise: Play with a pair of scissors. The blades are often slightly curved, but if you hold the blades close together, the tips of the blades are usually pretty straight. The gap between the blades forms an isosceles triangle with the two tips. Hold the tips of the blades close together, say half an inch apart. The separated blade length may be about an inch long. When you bring the tips together a distance of one half inch, the one inch gap closes. The joining of the blades is the phase wave. The joining of the tips is the group wave. There is a 90 degree relation between the tip's instantaneous motion and the blade's edge.

Exercise: Experiment with various similar triangles. The general rule is that they must be similar and they must share two different sides in common. Right triangles are standard, but other types of triangles are simply distorted versions of the standard. For example, similar triangles [A_1, B_1, C_1] and [A_2, B_2, C_2] might have $C_1 = A_2$ as their equal sides. Try making pairs of similar triangles according to this rule and play with them. See what system models you may discover.

Exercise: The Golden Ratio in a Golden Rectangle is an example of the Velocity Equation expressed spatially.



The above drawing shows an approximate Golden Rectangle. The larger rectangle is made from a square plus a smaller golden rectangle. The smaller one is made from a square and an even smaller golden rectangle. You can continue in this fashion making larger or smaller rectangles, and the pattern forms a Phi Spiral. The sides of the rectangles have the ratio: (A / C) = (C / B). In other words, (A) (B) = (C)^2. This is another example of the Velocity Equation represented spatially. Of course A, B, and C are also the sides of right triangles that share the side C. The smaller triangle is turned 90 degrees so that its C and the larger triangle's C form the ends of the larger Golden Rectangle.

The $(C)^2$ actually describes the square portion of the larger rectangle. So if $C = 1$, $B = \varphi$, and $A = (\varphi - 1)$. Or, if $C = \varphi$, then $B = (\varphi + 1)$, and $A = 1$. Thus $(1 / (\varphi - 1)) = (\varphi / 1) = ((\varphi + 1) / \varphi)$, where $\varphi = 1.618 \ldots$, an irrational number called the Golden Ratio. This ratio is also represented as $\varphi = (.5)(1 + (5^{.5}))$. The square root of 5 is the diagonal of a 1x2 rectangle. We get that by drawing a diagonal from the midpoint of C to an opposite corner of the larger square. If we set C = 2 units, then the diagonal is the square root of 5. By rotating the diagonal so that it runs from the midpoint of C [on top of the square] on out beyond the square, we get the total length of the Golden Rectangle as $(1 + (5^{.5}))$.

For more information about the Golden Rectangle, and its remarkable properties, look up the entry by Weisstein, Eric W. "Golden Rectangle." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/GoldenRectangle.html.

See also the articles on "Golden Ratio" and "Golden Triangle". Other good articles can be found at various sites on the web.